SOLUTION METHODS FOR MULTIPLE OBJECTIVE DECISION MODELS

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"Necessity saves us from the embarrassment of choice."

Vauvenargues,

Reflections and Maxims, 1746.

CONTENTS

ABSTRACT

This thesis is concerned with an investigation of solution methods for continuous multiple objective decision models (MODM's). A number of different solution methods which have appeared in the literature are reviewed with an emphasis on the underlying concepts of the methods. The following chapter examines the solution of MODM's from the other side, namely the behavioural aspects of decision making. Having gained an appreciation of exactly how people do make decisions, the intent of the thesis is twofold. Firstly, to develop new solution methods which can accommodate the decision maker (DM) in whatever his or her particular decision strategy is. And secondly, it is to empirically examine four solution methods with respect to users' preferences among them. Of these four solution methods, three are among the most well known in the literature and all can cite practical application.

Two new solution methods have been developed. Both of these methods are based on a specific formulation of the MODM which is known as the maxmin formulation. The theory of the maxmin formulation is developed in Chapter 4. By using the Lagrange multipliers at the optimal solution, suitable pairwise tradeoff information can be presented to the DM. This forms the basis for the first solution method, which interacts with the DM as he or she progressively provides preference information. The other solution method makes use of a branch of Psychology called Social Judgement Theory and incorporates this into the solution method. This second method is especially applicable to the multiple DM situation.

In the empirical examination of solution methods it was found that one solution method was clearly preferred over the other three. The thesis concludes with a discussion of approaches for reducing the number of objectives in a MODM.

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The responsibility for any typing errors and omissions remains my own.

CHAPTER 1 INTRODUCTION

Man will always be faced with the need to make decisions, decisions which must be made in the midst of a complex environment. And most of these decisions will have multidimensional consequences; i.e., a number of different criteria will be simultaneously affected by the decision. Although decisions and their outcomes are necessarily quite complex, the actual decision making process is quite simple, and can be stated as follows.

1. There is a decision maker (DM) who seeks to achieve some goals.
2. The DM has to choose from among two or more alternative courses of action.
3. There is some "doubt" (in terms of goal achievement) as to which alternative is most preferred.

This doubt on the part of the DM will inevitably occur as he or she is faced with relevant, yet conflicting, goals or objectives. Consider a decision whether to work overtime tonight. A number of objectives are relevant; extra income, loss of leisure, goodwill (i.e., will the boss offer overtime again if I refuse?) and possible effects on the family relationship. Somehow the human DM evaluates this information and chooses a course of action. Almost always there will be more than just a single objective (or criterion[1]) to be considered.

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1. The term "objective" will be consistently used throughout instead of "criterion". Both terms are found in the literature, somewhat interchangeably.

In the early days of management science, and especially with regard to practical applications, the multiple objective nature of decision problems was largely avoided. Instead, predominantly single objective models were used, with the most common unit of measurement being dollar value. This single objective approach, usually in the context of a linear programming model, resulted in a large number of successful practical applications, which in turn stimulated greater research in this area. The early growth of single objective decision models is likely to have slowed developments in the area of multiple objective decision models (MODM).

Starr and Zeleny (1977) give a brief outline of the origins of MODM in the field of management science. This work began in the early 1950's. After initial contributions, the next major contribution was that of goal programming [Charnes and Cooper (1961)]. In goal programming, a multiplicity of objectives are reduced to a single objective by minimizing deviations of each objective from certain pre-specified target levels or goals. The following decade saw traditional utility extended to multiattribute utility theory, and with Johnsen's (1968) study on the multigoal nature of the firm, Starr and Zeleny (p.12) suggest that "multiple criteria decision making was firmly on its path."

A relative explosion in research, papers and applications has occurred from the early seventies. A significant contribution of multiple objective solution methodologies has been to require more active participation on the part of the DM in the process of determining acceptable solutions to the MODM. This change restores in part the imbalance which resulted from the almost feverish application of single objective models in the sixties, where very little DM participation was required.

Initially, most work on MODM presupposed an "ideal" DM who always acted according to his or her utility function. A number of solution methods could therefore demonstrate convergence to the solution of maximum utility and were often quite elegant in concept. More recently, however, a number of researchers have sought to develop solution methods which are more consistent with the actual decision making behaviour of a DM and which place less emphasis on the more traditional utility approach.

This thesis is concerned primarily with this later development. The MODM to be considered is where the decision alternatives are stated explicitly in the form of constraints, with special attention given to the linear decision model. Briefly, the thesis is structured as follows. Chapter 2 first discusses necessary terminology and then reviews a number of solution methodologies which have appeared in the literature. Chapter 3 examines some features of actual decision making behaviour and considers the implications of these for solution methods. In Chapter 4 the theory of a particular form of MODM is developed, which provides a basis for new solution methods. Chapter 5 details an experiment where four different solution methods are compared. The experiment is designed to give a measure of discriminatory assessment among the four methods, and also to compare the results with the conclusions of Chapter 3. In Chapter 6 the details of two new solution methods are given, along with an example. This concludes the main portion of the thesis. Chapter 7 discusses possible approaches for reducing the number of objectives in a MODM. Chapter 8 is the conclusion.

CHAPTER 2 MULTIPLE OBJECTIVE DECISION MAKING: INTRODUCTION AND REVIEW

2.Ø INTRODUCTION

This chapter is concerned with introducing necessary terminology and definitions for the multiple objective decision model (MODM). Using these definitions a number of solution methods for the MODM which have appeared in the literature will then be reviewed, with an emphasis on the underlying concepts of the methods rather than on precise mathematical detail. The first section will be concerned only with the single decision maker (DM) situation. The chapter concludes with a brief overview of group decision making and a synthesis of the methods reviewed.

It is the intention of this chapter to communicate the developments in MODM's and methods of solution, along with sufficient detail, in order to provide a foundation for the chapters to follow. Greater attention will be given to solution methods which will be used in the experiment of Chapter 5 and also to those which provide insight into the developments of later chapters.

2.1 DEFINITIONS AND TERMINOLOGY - THE MODM

The MODM can be stated mathematically in terms of continuous decision variables. The general form of the model is

$$\begin{aligned} &\text{"MAX"} \quad F(\underline{x}) = [f_1(\underline{x}), f_2(\underline{x}), \ldots, f_q(\underline{x})] \\ &\text{subject to} \quad \underline{x} \in X \qquad [1] \end{aligned}$$

where

$\underline{x}$ is an n-dimensional Euclidean vector, $\underline{x} = (x_1, x_2, \ldots x_n)$

X is the set of feasible decisions

$$X = \{\underline{x} : \underline{x} \in R^n \ , \ g_i(\underline{x}) \leq \emptyset \ , \ i = 1,2,\ldots,m\}$$

$F(\underline{x})$ is a vector of scalar valued objective functions defined on $\underline{x}$

and "MAX" is defined in Section 2.1.1 below.

The set of feasible solutions to [1] is described by m continuous functions of the decision variables. This is in contrast to multiple attribute decision making (as it is often referred to in the literature), where the set of feasible solutions consists of a countably small number of discrete alternatives. The location decision for the Mexico City Airport [de Neufville and Keeney (1972)], and Rietveld's (198Ø) study of eight development alternatives for the Maasvlakte area in the Netherlands, are both examples where a few well defined alternatives constitute the entire feasible set. Approaches for solving the multiple attribute decision problem differ considerably from those developed to solve the continuous MODM, and will not be discussed in this chapter. Hwang and Yoon (1981) provide a useful survey of solution methods for multiple attribute decision models.

2.1.1 A Definition of "MAX"

The existence of a unique optimal solution to [1] is unlikely, except in the trivial case where a solution $\hat{\underline{x}} \in X$ maximizes each and every objective $f_k(\underline{x})$, $k = 1,2,\ldots,q$. Since such a solution cannot usually be found, the term "MAX" does not retain its traditional meaning [Rosenthal (1982)]. For any $(\underline{x}^1, \underline{x}^2) \in X$, the relationship between $F(\underline{x}^1)$ and

$F(\underline{x}^2)$ is not simply "greater than", "less than" or "equal to", since comparisons are required to be made across different and often incommensurable objectives. (For if the objectives were commensurable, then [1] could be reduced to a single objective problem. This is the approach adopted by traditional cost benefit analysis where it is assumed that all relevant costs and benefits can be expressed in monetary terms.) The solution to the MODM is therefore a set of solutions, which are called efficient or non-dominated solutions.

2.1.2 Dominance and Efficient Solutions

A definition of dominance is given below.

for $(\underline{x}^1, \underline{x}^2) \in X$ $\quad \underline{x}^1$ dominates $\underline{x}^2$ if

$$f_j(\underline{x}^2) < f_j(\underline{x}^1) \quad \text{for some } j \in \{1,2,\ldots,q\}$$
$$f_k(\underline{x}^2) \leq f_k(\underline{x}^1) \quad \text{for all } k <> j \qquad [2]$$

The efficient or non-dominated set consists of all feasible solutions to [1] which are not dominated by any other feasible solutions in X. Let $N \in X$ be the set of efficient solutions. Then for any $\underline{\hat{x}} \in N$, it is not possible to move to another $\underline{\tilde{x}} \in N$ without decreasing at least one objective function value. Geoffrion (1968) has extended this definition of [2] to a "properly efficient solution", which requires that the marginal gain for any one objective must be bounded relative to marginal losses in the other objectives.

Considerable research effort has been directed to finding solution methods which ensure that only efficient solutions to [1] are generated; in fact in many MODM solution methods, the actual optimization involves nothing more than distinguishing between efficient and inefficient solutions. As will be seen from the literature review to follow, almost all MODM solution methods only consider efficient solutions; consequently the characterization of efficient solutions is of high priority. Kuhn and Tucker, in presenting necessary and sufficient conditions for solving the single objective optimization problem, also extended their work to the multiple objective case. Let π_i, $i = 1,2,\ldots,m$ be the Lagrange multipliers for each constraint of X and assume that the objective functions are concave and the feasible set X is convex. The necessary and sufficient conditions for $\underline{x}^* \in X$ to be efficient are

$$\begin{aligned} \pi_i g_i(\underline{x}^*) &= \emptyset \qquad , \; i = 1,2,\ldots,m \\ \sum_{k=1}^{q} w_k \operatorname{grad}(f_k(\underline{x}^*)) - \sum_{i=1}^{m} \pi_i \operatorname{grad}(g_i(\underline{x}^*)) &= \emptyset \qquad [3] \\ w_k &\geq \emptyset \qquad , \; k = 1,2,\ldots,q \end{aligned}$$

where

$$\operatorname{grad}(f_k(\underline{x})) = (\, df_k(\underline{x})/dx_1, df_k(\underline{x})/dx_2, \ldots, df_k(\underline{x})/dx_n \,)$$

An insightful derivation of these conditions can be found in Goicoechea et al. (1982, pp.44-45) following an approach of Zadeh. While [3] gives the necessary conditions for a point to be efficient, a much more useful characterization has been given by Soland (1979).

Let h be any function defined on R^q which is strictly increasing on any of its components. For $b \in R^q$ define

$$\begin{aligned} P(h,b) = \ &\text{Max} \quad h[F(\underline{x})] \\ &\text{s.t.} \quad \underline{x} \in X \qquad [4] \\ &\qquad\;\; F(\underline{x}) \geq b \,. \end{aligned}$$

If $\underline{x}^*$ is an optimal solution to P(h,b), then $\underline{x}^*$ is efficient. This characterization effectively encapsulates two of the major approaches for solving the MODM. The first is to optimize a composite objective function (usually an additive form) subject to the constraint set, while the second optimizes a single objective subject to constraints on the achievement of all other objectives. These two forms are detailed below.

$$\begin{aligned} \text{Max} \quad & \sum_{k=1}^{q} w_k f_k(\underline{x}) \\ \text{s.t.} \quad & \underline{x} \in X \\ & w_k \geq \emptyset,\ k = 1,2,\ldots,q \end{aligned} \qquad [5]$$

$$\begin{aligned} \text{Max} \quad & f_j(\underline{x}) \\ \text{s.t.} \quad & f_k(\underline{x}) \geq b_k\ ,\ k = 1,2,\ldots,q\ ,\ k \neq j \\ & \underline{x} \in X \end{aligned} \qquad [6]$$

Using Soland's characterization a <u>single</u> efficient solution can be generated by assigning values to the parameters w_k and b_k.

Alternatively, the MODM can be solved to find all efficient solutions; this is known as the vector maximum approach. However, since the efficient set contains an infinity of solutions, some clarification is necessary. The vector maximum approach has been developed for the situation where all constraints and objectives to [1] are linear, and it finds all efficient extreme point solutions (which are finite in number). It is then possible to describe the infinity of non-extreme efficient solutions in terms of linear combinations of these extreme point solutions.

2.1.3 Matrix of Extreme Solutions and the Ideal Point

The matrix of extreme solutions is given by

$$P = \begin{bmatrix} f_1(\underline{x}_1^*) & f_2(\underline{x}_1^*) & & & \\ f_1(\underline{x}_2^*) & f_2(\underline{x}_2^*) & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \cdot \\ & & & & f_q(\underline{x}_q^*) \end{bmatrix} \qquad [7]$$

where $\underline{x}_k^*$ is the optimal solution to

$$\begin{aligned} &\text{Max} \quad f_k(\underline{x}) \\ &\text{s.t.} \ \underline{x} \in X \end{aligned} \qquad [8]$$

The ideal solution $\underline{U} = (U_1, U_2, \ldots, U_q)$
$= (f_1(\underline{x}_1^*), f_2(\underline{x}_2^*), \ldots, f_q(\underline{x}_q^*))$
is given by the diagonal of P and represents the maximum possible achievement of each objective $f_k(\underline{x})$. The ideal solution is often used as a point of reference in MODM solution methods, where a distance measure is presented to the DM to indicate how "far" the current solution is from the ideal solution.

2.1.4 Decision Space and Objective Space

In MODM, a distinction is often made between decision space and objective space. Each solution $\underline{x} \in X$ can be represented in terms of the decision variables ($\underline{x} = (x_1, x_2, \ldots, x_q)$) or in terms of the objective function values of those variables ($F(\underline{x}) = (f_1(\underline{x}), f_2(\underline{x}), \ldots, f_q(\underline{x}))$). Solutions are generally presented to a DM in terms of objective space since it is, for almost any realistic MODM, of much lesser dimension than decision space and therefore the presented information can be more easily assimilated by the DM.

2.1.5 Tradeoffs

Tradeoffs are a commonly used concept in MODM and in essence they are the relative changes in objectives when moving from one feasible solution to another, i.e., they are a measure of the difference between two solutions in objective space. Haimes and Chankong (1979) make the following useful distinction. Consider two feasible solutions $(\underline{x}^o, \underline{x}^*) \in X$. Define $T_{kj}(\underline{x}^o, \underline{x}^*)$ as the ratio of change in f_k to change in f_j. Thus

$$T_{kj}(\underline{x}^o, \underline{x}^*) = [f_k(\underline{x}^o) - f_k(\underline{x}^*)] \;/\; [f_j(\underline{x}^o) - f_j(\underline{x}^*)]. \qquad [9]$$

Then T_{kj} is a <u>pairwise</u> tradeoff if all

$$f_p(\underline{x}^o) = f_p(\underline{x}^*) \;,\; p = 1,2,\ldots,n \;,\; p <> j,k \;.$$

And T_{kj} is a <u>total</u> tradeoff if

there exists at least one p such that $f_p(\underline{x}^o) <> f_p(\underline{x}^*)$.

Tradeoffs can also be expressed in terms of a direction of movement. Let $\underline{d}^* = \underline{x}^o - \underline{x}^*$ be the direction in moving from $\underline{x}^*$ to $\underline{x}^o$, and α be the distance moved in that direction, $\underline{x}^o = \underline{x}^* + \alpha\underline{d}^*$. Then the total tradeoff rate at $\underline{x}^*$ along $\underline{d}^*$ can be defined as

$$\begin{aligned} t_{kj}(\underline{x}^*, \underline{d}^*) &= \lim_{\alpha \to 0} \; T_{kj}(\underline{x}^* + \alpha\underline{d}^*, \underline{x}^*) \\ &= \text{grad}(f_k(\underline{x}^*)).\underline{d}^* \;/\; \text{grad}(f_j(\underline{x}^*)).\underline{d}^* \qquad [10] \\ &= df_k(\underline{x}^*) \;/\; df_j(\underline{x}^*) \qquad \text{for} \quad \underline{d}^* = d\underline{x}^* \end{aligned}$$

Some methods for solving [1] seek to choose a direction $\underline{d}^*$ such that only partial or pairwise tradeoffs are used, while others utilize the concept of the total tradeoff. This concept of sacrificing an amount of one objective to achieve more of another is central to MODM, especially with respect to interactive solution methods. (This concept is not as useful for multiple attribute decision problems because the tradeoffs are not continuous, but discrete.)

2.2 DEFINITIONS AND TERMINOLOGY - THE DM's PREFERENCES

Up to this point, no mention has been made of the role of the DM in finding a solution to the MODM. In the absence of any participation from the DM, the actual solution to the MODM is not a single solution, but rather a set of solutions. A value judgement (i.e., subjective information) is required from the DM before a single "best" solution can be found. For the case of a single DM, the MODM can be restated as

> "find an $\underline{x}^* \in X$ such that the most preferred values of $F(\underline{x}^*)$ are obtained."

Such a solution is subjective, depending on the relative preferences of the DM among the different objectives. This is in contrast to the single objective decision model where the single best solution can be found in the absence of any subjective information from the DM. (Excepting, of course, where there exist alternative optimal solutions).

Obviously subjective information is not necessary in order to find a single solution to [1]. For example, a single global criterion such as the minimization of squared deviations from the ideal solution could be used. However, since the intent of MODM solution methods is to find the most preferred solution, it is unlikely that any method which does account for the DM's preferences will achieve this.

2.2.1 Utility or Value Functions

Given the inherent (and necessary) subjectivity in the process of finding a most preferred solution, the nature of the DM's preferences has a large impact on both the method of solution and the actual solution values. According to classical economics, the preferences of a rational DM are those of a utility maximizer; i.e., a DM is able to search among the set of feasible solutions and choose that solution which provides the greatest satisfaction or utility. Utility theory therefore assumes that, for an individual DM, there exists a scalar measure of preference for each $\underline{x} \in X$ which is his or her utility function.

There are certain conditions which the DM's preferences must satisfy for a utility function to be defined on them. The DM must be able to express both consistent preferences and consistent beliefs, and these beliefs (what the DM thinks is going to happen) are to be independent of preferences (what the DM would like to happen) [Hogarth (1980, Chapter 4)]. Consistent preferences imply transitivity; i.e., if A is preferred to B and B to C, then A is preferred to C. Consistent beliefs imply that predictive judgements regarding the occurrence of events can be formulated as probabilities, which means that there exist lotteries for which certainty equivalents can be derived, e.g., Keeney and Raiffa (1976, pp.142-148).

Let $\underline{V}$ $[F(\underline{x})]$ be a utility function which represents the preferences of a DM (as scalar values) over the set of feasible solutions. The MODM can be reformulated to find the most preferred solution by solving

$$\begin{aligned} \text{Max} \quad & \underline{V}\ [f_1(\underline{x}), f_2(\underline{x}), \ldots, f_q(\underline{x})] \\ \text{s.t.} \quad & \underline{x} \in X \end{aligned} \qquad [11]$$

In [11] all objectives have been aggregated into a single scalar measure which is then optimized to find the solution $\underline{x}^*$ of maximum utility. The practical difficulty with this approach is the determination of a suitable form for $\underline{V}$.

In order to facilitate the assessment of the overall utility function $\underline{V}$, decomposition forms are often used, i.e.,

$$\begin{aligned} & \underline{V}\ [f_1(\underline{x}), f_2(\underline{x}), \ldots, f_q(\underline{x})] \\ & \qquad = H(V_1[f_1(\underline{x}), V_2[f_2(\underline{x})], \ldots, V_q[f_q(\underline{x})]). \end{aligned} \qquad [12]$$

A simple and commonly used form for H is the weighted additive form, which for three objectives can be stated as

$$\begin{aligned} & \underline{V}[f_1(\underline{x}), f_2(\underline{x}), f_3(\underline{x})] \\ & \qquad = w_1 V_1[f_1(\underline{x})] + w_2 V_2[f_2(\underline{x})] + w_3 V_3[f_3(\underline{x})] \end{aligned} \qquad [13]$$

where w_k , $k = 1,2,3$ measures the relative contribution of each objective. Zeleny (1982, p.418) lists some other common decomposition forms. The "cost" of using decomposition forms is that a further two conditions are placed on the DM's preference structure. These are preferential independence, where the value of a tradeoff between any two objectives is independent of the level of a third objective; and utility independence where the DM's preferences for lotteries on one objective are independent of the level of a second objective. In practice, it is assumed that these conditions hold, because unless some simple decomposition form is used, assessment of the DM's utility function is almost impossible.

2.2.2 Marginal Rate of Substitution

In contrast to the tradeoffs previously mentioned in Section 2.1.5, the marginal rate of substitution (MRS) is the value of a tradeoff according to a DM's utility function, rather than according to the geometry of the feasible set X. The MRS is a pairwise tradeoff, not a total tradeoff. With respect to a utility function $\underline{V}$, MRS_{kj} is defined as the amount of f_k that a DM is willing to sacrifice to acquire an additional unit of f_j at any given point $\underline{f}^o$ in objective space, i.e.,

$$\begin{aligned} MRS_{kj}(\underline{f}^o) &= (\delta \underline{V}\,[\underline{f}^o]/\delta f_j) \;/\; (\delta \underline{V}\,[\underline{f}^o]/\delta f_k) \\ &= -\,df_k/df_j \quad \text{for fixed utility.} \end{aligned} \qquad [14]$$

Figure 2.1 on the following page shows both tradeoff and MRS values. This figure shows the set of feasible solutions in objective space (only two objectives) with the DM's utility function superimposed on top for certain fixed levels of utility. The efficient set is defined by the line segments BC and DE and the ideal point is $(f_1^*, f_2^*) = (10, 6)$. D is the feasible (and efficient) solution of maximum utility where MRS is equal to the pairwise tradeoff.

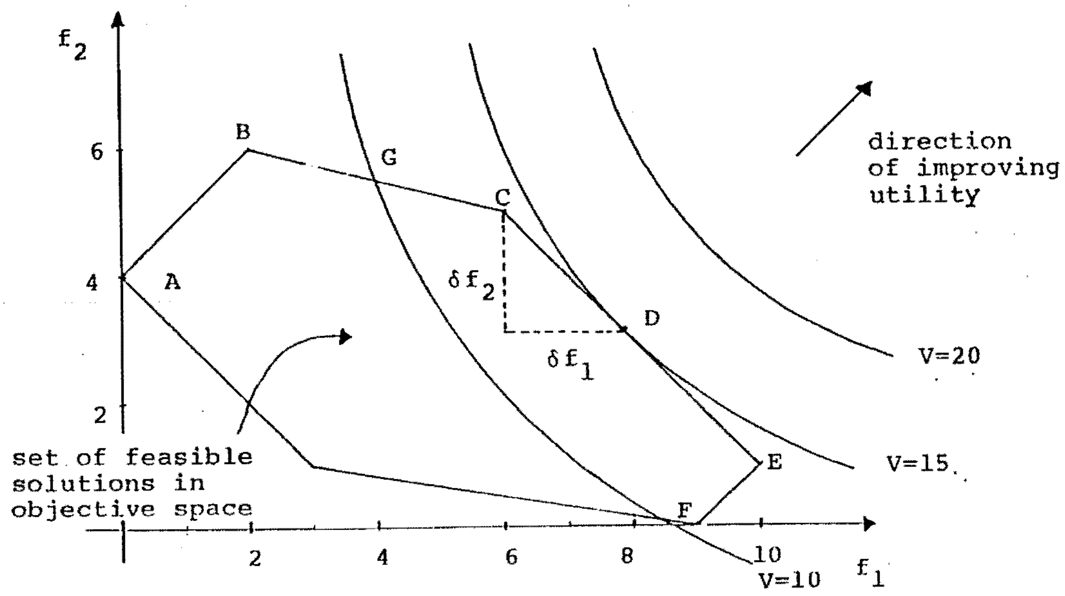

Figure 2.1

At D $\quad MRS_{21} = (\delta \underline{V}/\delta f_1) \; / \; (\delta \underline{V}/\delta f_2) = 1$

$\quad\quad t_{21} = - \; \delta f_2/\delta f_1 \quad\quad\quad = 1$

At G $\quad MRS_{21} = 2.83 \quad\quad$ and $\quad\quad t_{21} = 0.25$

This MRS value indicates that, at solution G, the DM is willing to sacrifice up to 2.83 units of f_2 to gain an additional unit of f_1, whereas the tradeoff value at G says that an additional unit of f_1 can be obtained by sacrificing only 0.25 units of f_2. There are obviously better solutions than G.

2.2.3 Monotonicity of Preferences

A further assumption which is usually made in the context of the MODM is that the DM's preferences are a monotone function of the level of each objective function. A DM is assumed to always prefer more to less; his or her satisfaction does not decrease as f_k , $k \in \{1,2,...,q\}$ increases. The marginal value of an extra unit of objective f_k , $k \in \{1,2,...,q\}$ is always greater than or equal to zero.

The consequence of this assumption is that the DM's most preferred solution will always be efficient. This assumption also accounts for the large emphasis which has been placed on generating efficient solutions to the MODM [1]. In general, the monotonicity of preferences assumption is a reasonable one, especially when care is taken to appropriately define the objectives. As a contrary example, consider an objective in a financial MODM which is to achieve a current ratio with a value of 2. For this objective, a DM's preferences are likely to be represented by the curve in Figure 2.2 below. Maximization of the current ratio is an inappropriate objective; a more realistic objective for which preferences are monotone would be to minimize deviations from the ideal value of 2.

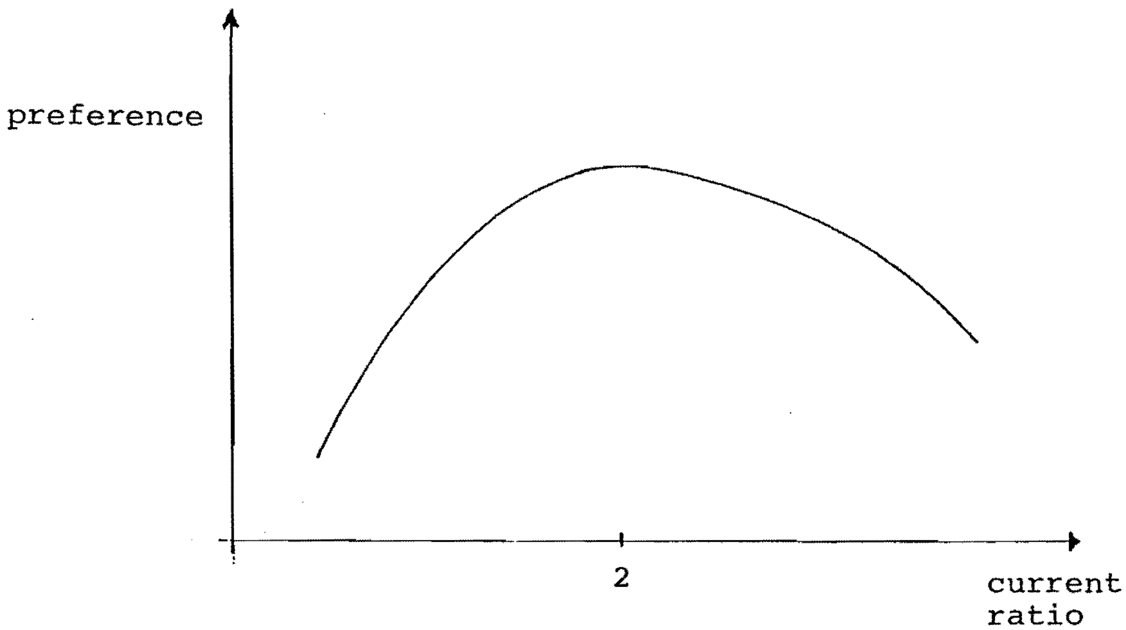

Figure 2.2

This concludes the introductory terminology and definitions as relevant to the MODM and necessary for the following discussion.

2.3 METHODS OF SOLUTION

Some of the solution methods for MODM's which have appeared in the literature will now be discussed. For other more detailed reviews see Hwang and Masud (1979) or Chankong and Haimes (1983a).

Multiple objective solution methods vary according to the characteristics of the problem formulation (e.g., linear or non - linear and size) and according to the information provided by the DM. Hwang and Masud (1979) provide a useful classification of methods according to the timing of the information provided. Their three categories are

a priori - before solution
progressively - during solution (interactive methods)
a posteriori - after solution.

A different approach is taken by Ho (1981) who proposes a hierarchical classification according to the quantity of information provided. This section will follow the classification of Hwang and Masud and consider only the single DM situation. Also the emphasis will mainly be on the linear MODM, as this has been the focus of most research. The review of solution methods will not consider any methods in great detail; rather it is intended to present underlying concepts of the major solution methods. This is to demonstrate the various solution approaches and also to provide sufficient background for the discussion of the following chapter.

2.3.1 A PRIORI ARTICULATION OF PREFERENCES

The solution methods in this category first elicit subjective information from the DM which is then utilized to find a preferred solution.

2.3.1.1 Goal Programming

Before the advent of multiple objective solution methods, the method of solution usually consisted of maximizing a single objective with all other objectives constrained to certain acceptable or satisfactory levels. Goal programming (GP) was the first truly multiple objective solution method to be developed [Charnes and Cooper (1961)], and is based around the intuitive concept of goal setting. Specifically, the DM assigns a goal or target to each objective and then seeks to minimize the deviations from each goal. These deviations, which represent both over and under achievement of goals, are then weighted by the DM so as to reflect their relative importance. The GP formulation of [1] is

$$
\begin{aligned}
\text{Min} \quad & \sum_{k=1}^{q} \left(w_k^- d_k^- + w_k^+ d_k^+ \right) \\
\text{s.t.} \quad & f_k(\underline{x}) + d_k^- - d_k^+ = G_k \quad , \quad k = 1,2,\ldots,q \\
& \underline{x} \in X \\
& d_k^- \, , \, d_k^+ \geq \emptyset \quad , \qquad k = 1,2,\ldots,q
\end{aligned}
\qquad [15]
$$

where G_k is the goal for objective k and d_k^+ and d_k^- are measures of over and under achievement respectively.

The deviational weights w_k^+ and w_k^- can be either pre-emptive, in which case goals of higher priority must be fully satisfied before lower priority goals are considered;

or additive, whereby all goals are at the same priority level and considered simultaneously. Combinations of these two cases are also possible, e.g., using additive weights among three objectives which all have the same priority. The informational burden placed on the DM is to specify two types of information for each objective before solution. These are the goals G_k to be attained and the weights w_k.

A number of similarities can be seen between GP and the first high level computer language, FORTRAN. Both represent pioneering efforts in a particular area and have now become well established and widely used. In the same way that, as more computer languages have been developed, FORTRAN (in its original form) has come under much criticism, goal programming has also faced not inconsiderable criticism. This criticism includes the substantial burden placed on the DM to provide realistic goals before solution, the possibility of generating inefficient solutions [Zeleny (1982, pp.296-298)] and the validity of tradeoffs under a pre-emptive weighting structure. As Rosenthal (1983) points out, the use of pre-emptive or priority weights is contrary to the concept of a MRS. Pre-emptive weights imply that since one objective f_j must be fully satisfied before a second objective f_k is even considered, the MRS_{kj} is infinite. No amount of inducement will convince the DM to sacrifice some of f_j to gain some of f_k; a result which is contrary to intuition.

However, when compared with other MODM solution methods, GP can attest to a wealth of applications, [e.g., Lin (1980)]. While this is in part due to its position as a pioneering solution method, the widespread use of GP can also be attributed to the intuitive and readily understood concept of setting goals and trying to get as close as possible to them. Furthermore, GP can accommodate certain decision situations which are more difficult with other solution methods. For example, Sartoris and Spruill (1974) use GP in a financial decision model where objectives are not always linear and preferences are not monotonic for given objectives. In their working capital model one objective is to achieve a liquidity ratio of one,

i.e., $$(x_8 + 40x_2 + 52.5x_4) / (150 + x_7) = 1 .$$

This can be multiplied through and deviational variables added to give

$$x_8 + 40x_2 + 52.5x_4 - x_7 + d^- - d^+ = 150$$

where minimization of the deviations will seek to achieve the desired liquidity ratio. An alternative solution approach would be to use fractional objectives [e.g., Choo and Atkins (1980)].

Unlike many other MODM solution methods which attempt to converge to the DM's most preferred solution, GP is more a mechanism for generating solutions which will reflect the goals and weights specified by the DM. Consequently, there is no guarantee with GP that the most preferred solution will be found.

It is not surprising to find that GP methods have been extended to interact with the DM in order to relieve some of the burden of a priori information provision and to provide a more systematic approach in searching for the most preferred solution, [e.g., Dyer (1972), Masud and Hwang (1981)].

2.3.1.2 Surrogate Worth Tradeoff (SWT) Method

This method is based on the previously mentioned characterization of an efficient solution [6], which is known as the e-constraint formulation. It is

$$\begin{aligned} &\text{Max} \quad f_j(\underline{x}) \\ &\text{s.t.} \quad f_k(\underline{x}) \geq e_k \quad , \; k = 1,2,\ldots,q \; , \; k \neq j \\ &\qquad \underline{x} \in X \; . \end{aligned} \qquad [16]$$

Developed by Haimes and Hall (1974), SWT utilizes the concept of a pairwise tradeoff between two objectives. It can be shown that for F differentiable and λ_{jk} being the Lagrange multiplier for constrained objective k, the pairwise tradeoff t_{jk} is defined as

$$t_{jk} = df_j \,/\, df_k = -\lambda_{jk} \qquad [17]$$

In the SWT method the DM is required to assign a value to various pairwise tradeoffs which are presented to him or her. Specifically, for different efficient solutions the DM assigns a value w_{ij} to each pairwise tradeoff λ_{ij} for $i,j \in \{1,2,\ldots,q\}$. The preferred solution will be where $w_{ij} = \emptyset$ for all i,j, i.e., the point of indifference. Using the following relationships,

$$\lambda_{ij} = 1/\lambda_{ji} \quad , \quad \lambda_{ik} = \lambda_{ij}\lambda_{jk} \qquad [18]$$

all pairwise tradeoffs can be calculated for any properly efficient solution to [16]. (At any improperly efficient solution, some λ_{ij} , $i \in \{1,2,..,q\}$, $i \neq j$ will be zero.) w_{ij} are called the surrogate worth functions and are ordinal in nature. If $w_{ij} > \emptyset$ then it is assumed that $MRS_{ij} > \lambda_{ij}$

(the tradeoff is favoured, i.e., decrease f_i and increase f_j). For $w_{ij} < 0$, the reverse applies. After the DM has provided this information at a number of solutions, a solution in objective space is determined whereby $w_{ij} = 0$ for $i = 1,2,\ldots,q$, $i <> j$. The solution values at this point of indifference are used to constrain the objectives in [16], which is then solved to find the most preferred value of the remaining objective f_j. This solution will then be the most preferred solution, where the DM has provided these worth assessments <u>prior</u> to solution.

Often it is not specified exactly how the indifference solution (where <u>all</u> w_{ij}'s are zero) is calculated. If in the course of the evaluation, one such indifference solution is found, then there are no difficulties. However this is generally not the case. When there is no point of indifference immediately obvious from the w_{ij} values, the most commonly recommended approach is to use multiple regression. q-1 regressions are performed of the form

$w_{kj} = w_k(f_1, f_2, \ldots, f_q)$ for each $k \in \{1,2,\ldots,q\}$, $k <> j$

where w_{kj} are the worth assessments for the pairwise tradeoff between f_k and f_j. Setting all the $w_{kj} = 0$ gives a set of q-1 simultaneous equations which can be solved to find the indifference solution with values $(f_1^*, f_2^*, \ldots, f_q^*)$. These values are substituted into [16] which is then solved to find f_j^*. On the basis of a small amount of experimentation, it was found that this multiple regression approach often gave unsatisfactory results. For the experiment of Chapter 5, a somewhat different approach was used following Goicoechea et al. (1982, pp.143-149). Details of this approach are provided in Section 5.2.3 .

According to Haimes et al.(1975, p.34), the SWT method is based on the fact that "optimization theory is usually more concerned with the relative value of additional increments of the various non-commensurable objectives, at a given value of each objective function, than it is with their absolute values". This, coupled with the observation that it is generally easier for a DM to provide pairwise tradeoffs than total tradeoffs, provides the motivation for the method.

As GP methods have been made interactive, so too has the SWT method. Chankong and Haimes (1978) describe the interactive surrogate worth (ISWT) method which will also be mentioned in Section 2.3.2.2.3 .

GP and the SWT method, along with utility function assessment, are the main solution methods where information is provided by the DM prior to solution. The extension of these two methods to an interactive form is likely to be indicative of the unsuitability of a priori information provision.

2.3.2 PROGRESSIVE ARTICULATION OF PREFERENCES

The majority of MODM solution methods belong in this second category. The intent of these methods is that via interaction and progressive revelation of preferences, a sequence of solutions will result. This sequence of solutions should in the limit converge to the most preferred solution of the DM.

2.3.2.1 STEM Method

The STEM method, proposed by Benayoun and his colleagues (1971) for linear MODM's, was one of the first interactive solution methods to be developed. It is conceptually simple, with the preferences of the DM being implicitly incorporated into the solution method by setting bounds on the objectives. The formulation combines [6] with a variation of [5] with the DM being required to progressively provide values for b. The parameters of the function h are determined endogenously and are a form of weighted distance metric from the ideal solution. At the first iteration the following problem is solved.

$$\begin{aligned} &\text{Min} \quad y \\ &\text{s.t.} \quad y \geq (U_k - f_k(\underline{x}))w_k \ , \ k = 1,2,\ldots,q \\ &\quad \underline{x} \in X \end{aligned} \qquad [19]$$

where U_k and w_k are calculated from the extreme solution matrix. The U_k values are found from the diagonal elements and the w_k values are calculated as

$$w_k = \alpha_k \Big/ \sum_{k=1}^{q} \alpha_k \ , \quad \alpha_k = [(U_k - M_k)/M_k] \Big/ [\sum_{j=1}^{n} (c_{jk})^2]^{1/2}. \qquad [20]$$

M_k is the minimum value of each column k of the extreme solution matrix and c_{jk} are the coefficients of each linear objective function, $f_k(\underline{x}) = \sum_{j=1}^{n} c_{jk}x_j$. w_k can be interpreted as a measure of the relative discrepancy between the maximum and minimum values of $f_k(\underline{x})$. A solution $\hat{\underline{x}}$ to [19] is presented to the DM in terms of the objectives. The DM is then required to specify an amount by which a satisfactory objective f_j is to be relaxed in order to allow improvement in the other objectives. Therefore, at each iteration, the constraint set is augmented by a constraint of the form

$$f_j(\underline{x}) \geq f_j(\hat{\underline{x}}) - \delta_j \qquad [21]$$

where δ_j is the amount of relaxation. w_j is then set to zero and the next iteration begins. Since the DM sets bounds for a different satisfactory objective at each iteration, the procedure should terminate after q-1 iterations with either the most preferred solution or the message that there is no solution acceptable to the DM.

A number of extensions to this approach have been proposed. These include Belenson and Kapur (1973) who use a two person zero sum game approach to determine the appropriate weights at each iteration, and a goal programming extension of the STEM method [Fichefet (1974)]. Nijkamp and Rietveld (1976) suggest that the weights at each iteration can be chosen such that each solution of the extreme solution matrix is valued equally. In matrix notation, the appropriate weights are

$$\underline{w} = [\ (P^T)^{-1}I\ ] \ /\ [\ I^T(P^T)^{-1}I\ ]\ , \qquad [22]$$

provided the extreme solution matrix P is non-singular. A disadvantage with this approach is that negative weights may be generated. And because Nijkamp and Rietveld use an objective of the form $\underline{w}.C$ (where C is the matrix of objective function coefficients), the use of negative weights may result in inefficient solutions being generated at some iterations.

2.3.2.2 Method of Geoffrion, Dyer and Feinberg (GDF) (1972)

In contrast to the STEM method, the GDF method places a greater informational burden on the DM, in that he or she is required at each iteration to provide a MRS value for any pair of objectives. The method assumes that, at least implicitly, the DM possesses a utility function defined on the q objective functions. The MODM therefore becomes that of [11]

$$\text{i.e.,} \quad \text{Max } \underline{V}\,[F(\underline{x})] \qquad \text{s.t. } \underline{x} \in X \,. \qquad [23]$$

[23] is solved by utilizing the Frank-Wolfe algorithm, which itself uses linear approximations to the utility function at each iteration. There are two steps at each iteration; finding a best direction of improvement and finding a best step size in that direction. At a feasible (although not necessarily efficient) solution $\underline{x}^i$, the direction finding problem for [23] reduces to

$$\text{Max} \sum_{k=1}^{q} w_k(\underline{x}^i)\ \text{grad}(f_k(\underline{x}^i)).\underline{y} \qquad \text{s.t. } \underline{y} \in X \qquad [24]$$

where $w_k(\underline{x}^i)$ = MRS between f_k and an arbitrary reference objective f_j. The neat thing about this method is that the exact form of the DM's utility function is not required. Provided that the DM is able to give MRS information at each iteration, which is consistent with his or her utility function, the direction of best improvement $\underline{d}^i$ can be found. In order to find the step size α, various values of

$$F(\underline{x}^i + \alpha\underline{d}^i) \quad \text{for} \quad 0 \leq \alpha \leq 1 \qquad [25]$$

are presented to the DM, who chooses the most preferred one. The next feasible solution $\underline{x}^{i+1} = \underline{x}^i + \alpha\underline{d}^i$ is found and the iterations continue.

The sequence of solutions which can begin at any feasible solution, will not necessarily be efficient until the final, most preferred solution is reached. As regards possible disadvantages of the method, Hwang and Masud (1979, p.121) comment that the DM often has difficulty in providing MRS information at each iteration and where there are more than two objectives, the choice of a suitable reference objective can also be difficult.

A number of other solution methods which are similar in spirit to the GDF approach and yet exhibit further extensions in concept will also be briefly reviewed.

2.3.2.2.1 Efficiency Projections - Winkels and Meika (1984)

This approach generates only efficient points at each iteration. Once the direction of best improvement has been found from the MRS information of the DM, it is projected onto the efficient surface. This information is presented graphically to the DM who then chooses the appropriate step size. Figure 2.3 (on the following page) which is taken from the article, gives an example of the efficiency projections for a single iteration. If the ordinary linear approximation of the GDF method is used, the result is a straight line between the endpoints of $\lambda = 0$ and $\lambda = 1$, as illustrated by objectives 2 and 4 in the diagram. The critical values of λ, where the direction of the efficient projection changes, represent a change in basis. The form in which the information is presented provides considerable insight into the total tradeoffs in a given direction.

The method is exactly that of the GDF method, except that the initial and all subsequent solutions are efficient.

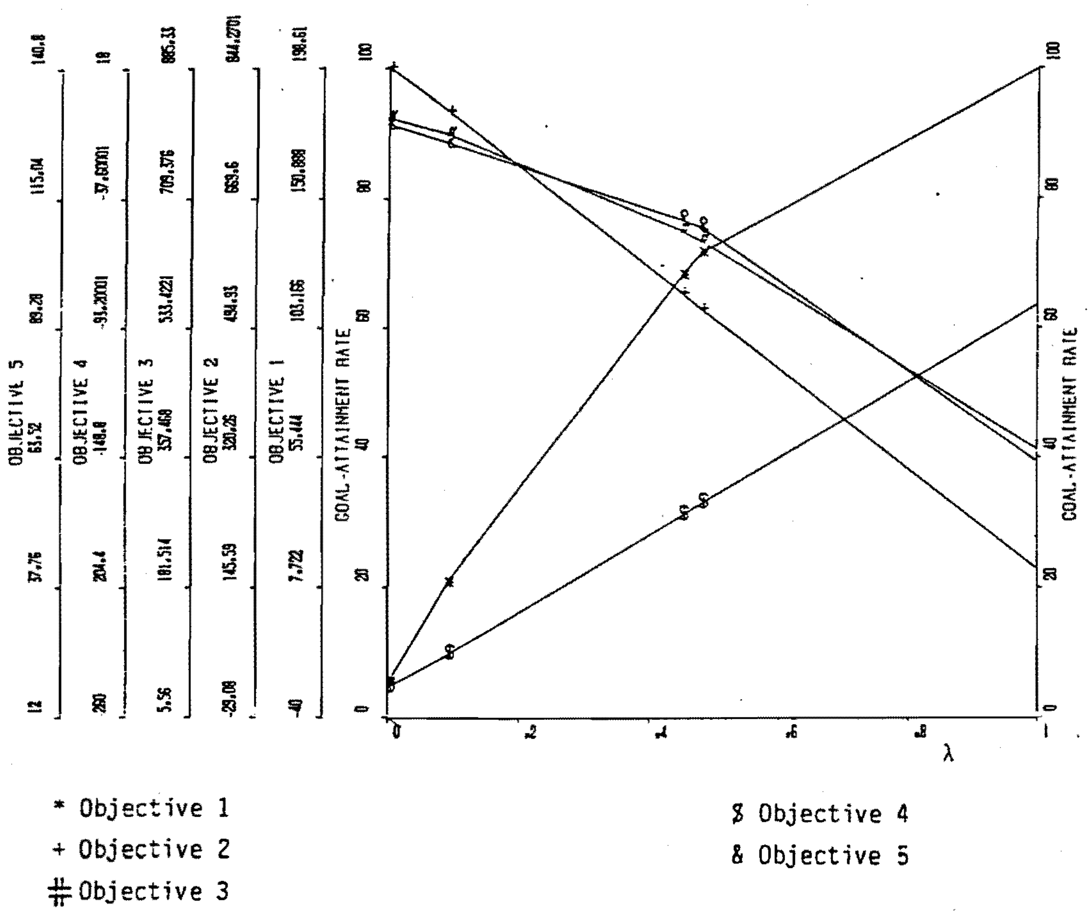

Figure 2.3

2.3.2.2.2 Proxy Approach - Oppenheimer (1978)

Where the GDF method uses linear approximations to the DM's true utility function at each iteration, Oppenheimer makes use of an assumed proxy utility function to do the approximating. The MRS information provided at each iteration by the DM is sufficient to evaluate the parameters of an assumed proxy utility function. Given this, it is reasoned that a proxy utility function should be a better local approximation to the DM's true utility function than a linear approximation as used by GDF. The proxy utility functions are not intended to be globally valid; in fact a different proxy will be evaluated at each iteration. Two standard form proxy utility functions used by Oppenheimer are

$$\text{sum-of-exponentials} : \underline{V}[F] = -\sum_{k=1}^{q} a_k \exp(-w_k f_k) \qquad [26]$$

$$\text{sum-of-powers} : \underline{V}[F] = -\sum_{k=1}^{q} a_k (M_k+f_k)^{\alpha_k} , \; M_k+f_k > \emptyset$$

The choice as to which form of proxy function is actually used is at the discretion of the analyst. This proxy approach has a considerable advantage over the GDF method since both the best direction of improvement and the step size are calculated from the local proxy utility function at each iteration.

2.3.2.2.3 SPOT Method - Sakawa (1982)

It is necessary to briefly introduce the interactive surrogate worth tradeoff (ISWT) method [Chankong and Haimes (1978)] in order to illustrate the SPOT method. The ISWT method is an interactive scheme based on the e-constraint formulation [16]. It uses Zoutendijk's method of steepest descent to determine a direction of improvement, which is found from the worth values provided by the DM, and a step size at each iteration. At a given solution $\underline{x}^i$, surrogate worth values w_{jk} are assigned by the DM to each tradeoff λ_{jk}. The updated right hand side of each constrained objective k is simply given by

$$e_k^{i+1} = e_k^i + \alpha^i (w_{jk} : f_k(\underline{x}^i)) \qquad [27]$$

where α is the step size. The similiarity with the GDF method can be easily seen; the w_{jk} values contain the necessary MRS information. This procedure will converge to the most preferred solution under the assumption of an ideal DM (who conforms to his or her implicit utility function).

The SPOT method uses the same basic approach as the ISWT method except that (like Oppenheimer) a proxy utility function is used to determine the best direction of improvement. Specifically, if m^i_{jk} is the MRS_{jk} based on the DM's proxy utility function at solution $\underline{x}^i$, then the updating equation is

$$e^{i+1}_k = e^i_k + \alpha^i(m^i_{jk}-\lambda_{jk}) \qquad [28]$$

Both SPOT and Oppenheimer's proxy approach require consistency checks to be made of the DM's preferences with regard to the particular proxy function which is being assessed. If discrepancies exist beyond a certain prespecified tolerance level, the inconsistency is explained to the DM so that the tradeoffs can be reassessed and the discrepancy resolved.

Sakawa and Seo (1983) have further extended the SPOT method by using fuzzy set theory. They assume that the DM has a local, but imprecise knowledge of his or her utility function. This means that instead of having to specify an exact value for each MRS assessed, one approach is to require the DM to provide four values. These four values are the absolute minimum and maximum for the MRS and the minimum and maximum of a totally acceptable interval for the MRS. This information is processed using the theory of flat fuzzy numbers. Here an attempt is made to accommodate some of the imprecision expected from a DM, and therefore there must also be some tolerance of inconsistency.

2.3.2.2.4 Tchebycheff Norm Approach - Sakawa and Mori (1983)

Instead of using the e-constraint formulation as in the SPOT method, in this method Sakawa and Mori use a weighted Tchebycheff norm formulation. (This Tchebycheff formulation will be discussed in considerable detail in Chapter 4.) A direction of improvement $\underline{d}$ is found using MRS information provided by the DM. With a step size parameter of λ, solutions in this direction ($\underline{f}(\underline{x}^*) + \underline{\lambda d}$) are then found, except that these solutions are first transformed into efficient solutions by using the Tchebycheff formulation. This approach is similar in principle to that of Winkels and Meika (1984) where all solutions in the direction of improvement are projected onto the efficient surface. The difference lies with the actual projection mechanism.

This section has reviewed a number of extensions to the original GDF method. Given all these variations, there is the potential to combine some of these approaches to arrive at a useful solution methodology.

2.3.2.3 Method of Zionts and Wallenius (ZW) (1976,1982)

In order for this method to be of practical use for solving the MODM [1], it is required that all functions be linear. Furthermore it is assumed that the DM's underlying or implicit utility function is a linear combination of the objectives. Thus the problem to be solved is

$$\text{Max } \underline{V} = \sum_{k=1}^{q} w_k f_k(\underline{x})$$
$$\text{s.t. } \underline{x} \in X \qquad [29]$$

$$\sum_{k=1}^{q} w_k = 1 \quad , \quad w_k \geq \emptyset \quad , \quad k = 1,2,\ldots,q. [3\emptyset]$$

At each iteration a set of weights $\underline{w} = (w_1, w_2, \ldots, w_q)$ is determined until the process terminates with the weighting structure which maximizes the linear utility function. The most preferred solution will be an extreme point of the efficient set because the composite objective in [29] is linear. Specifically, the method operates as follows. An arbitrary set of weights which satisfy [3Ø] are chosen (usually equal weights in the absence of other information) and the resulting solution $\underline{x}^p$ is found. Then for each non-basic variable x_r^N the following subproblem test is performed.

$$\text{Min } f_r = \sum_{k=1}^{q} v_{kr} w_k \qquad [31]$$

$$\text{s.t. } \sum_{k=1}^{q} v_{kj} w_k \geq \emptyset \text{ , } j \in \{j : x_j \text{ is non basic}\}, \; j <> r$$

Let f_r^* be the optimal solution to [31]. If $f_r^* < \emptyset$, then introducing x_r^N into the basis will result in moving along an efficient edge to another efficient solution. Zionts and Wallenius also implement some "quick check" rules such that some non-basic variables are immediately eliminated from consideration, which reduces the necessary computational effort. The v_{kj} values are the Lagrange multipliers from the optimal simplex tableau to [29] and [3Ø] for objective k and non-basic column j. Effectively, $\underline{v}_j = (v_{1j}, v_{2j}, \ldots, v_{qj})$ is the total tradoff vector which results if x_j^N is pivoted into the basis to give an adjacent efficient solution. The DM is required to provide an ordinal response of "yes", "no" or "indifferent" to each total tradeoff $\underline{v}_j$.

The burden placed on the DM as regards providing information is affected in two ways. It is lessened in that only ordinal rather than cardinal information is required, but is increased in the sense that the DM has to assess the tradeoff holistically, i.e., over all objectives simultaneously. From the DM's responses, constraints are constructed as follows

$$\text{"yes"} : \quad \sum_{k=1}^{q} v_{kj} w_k \leq -e$$

$$\text{" no"} : \quad \sum_{k=1}^{q} v_{kj} w_k \geq e \quad , \quad 0 < e \ll 1 \qquad [32]$$

with "indifferent" responses being omitted for reasons of accuracy and speed of convergence. A "yes" answer implies that the DM prefers the tradeoffs in the direction of an adjacent solution $\underline{x}^{p+1}$, therefore it must have greater utility than the old solution $\underline{x}^p$.

$$\text{i.e.,} \quad \underline{V} = \sum_{k=1}^{q} (-v_{kj}) w_k \geq e \qquad [33]$$

$$\text{since} \quad v_{kj} = f_k(\underline{x}^{p+1}) - f_k(\underline{x}^p) = \delta f_k$$

After responses to every tradeoff vector $\underline{v}_j$, a feasible set of weights $\underline{w}^2$ which satisfy [32] are found. These weights are used to solve [29] and [30], as the next iteration of the method begins. A new set of tradeoffs are presented to the DM and the process continues with each response of the DM being appended to the set of constraints

[32]. (When subsequent subproblem tests [31] are performed the constraint set of [32] is also appended to ensure that adjacent efficient extreme points, which are not consistent with previous responses, will be excluded). The process terminates when there is only one efficient extreme point which is consistent with the previous responses of the DM.

Effectively, the ZW method cuts away a portion of the objective space at each iteration, with the advantage that many solutions can be implicitly eliminated. De Samblanckx et al.(1982) have observed that in this method one wrong answer is irrevocable and that tradeoffs are not always fulfilled exactly how they are presented to the DM. Since this method is included in the experiment of Chapter 5, some of these issues will become clearer as an example of the method in operation is given.

More recently, Stewart (1984) has proposed a modification to the ZW method whereby provision is made for inconsistent choice behaviour. The problem of finding the vector of weights $\underline{w}$ at each iteration is approached by maximizing the following log likelihood function.

$$L(\underline{w}:S_n) = - \sum_{j \in S_n} \log \left[1 + \exp\left(- \sum_{k=1}^{q} w_k v_{kj}\right) \right] \qquad [34]$$

where S_n is a set of pairwise preference statements given by the DM. While there are some additional features to Stewart's logistic regression approach, the basic concept is to allow for inconsistencies by using maximum likelihood estimation. However, despite Stewart's own comment that "the Zionts-Wallenius method is quite often relatively insensitive to response errors" (p.1077), his extension to the method represents another attempt to reduce the requirements placed on the DM as information is progressively elicited from him or her.

2.3.2.4 Method of Interval Criterion Weights and its Extensions

Again it is assumed that the MODM is linear. This method [Steuer (1977)] derives from methods for generating the entire efficient set (see Section 2.3.3) and earlier work of Steuer (1976). In this earlier work, Steuer showed how the size of the efficient set can be reduced if the DM is able to a priori specify bounds on the weights as in the following weighted sum formulation.

$$\begin{aligned} \text{Max} \quad & \sum_{k=1}^{q} w_k f_k(\underline{x}) \\ \text{s.t.} \quad & \underline{x} \in X \\ & \sum_{k=1}^{q} w_k = 1 \\ & w_k \in (m_k, u_k) \quad , \quad \emptyset \leq m_k \leq u_k \leq 1 \end{aligned} \qquad [35]$$

[35] cannot be solved in its current form; however, Steuer shows that it can be reduced to

$$\begin{aligned} \text{Max} \quad & D(\underline{x}) = [d_1(\underline{x}), d_2(\underline{x}), \ldots, d_r(\underline{x})] \\ \text{s.t.} \quad & \underline{x} \in X \end{aligned} \qquad [36]$$

where d_j, $j = 1,2,\ldots,r$ represent the extreme rays of the reduced gradient cone as defined by the bounds m_k and u_k on the weights. (The gradient cone or cone of "good directions" is the convex cone generated by the gradients of the different objectives.) Each extreme ray d_j is defined by critical weights w_{kj}^*

i.e., $$d_j = \sum_{k=1}^{q} w^*_{kj} f_k \ , \ j = 1,2,\ldots,r \qquad [37]$$

where the critical weights are determined by feasible combinations of the interval endpoints (m_k, u_k). For a given set of interval bounds, it is not possible to determine in advance the number of extreme rays r required to define the reduced gradient cone. The gradient cone can perhaps best be visualized (in objective space) by imagining a light source at the origin which shines on the efficient surface. Initially the cone of light illuminates the whole surface and it is narrowed down as interval bounds are specified for the weights.

It is this process of narrowing down the light cone which Steuer has developed into an interactive method. In terms of the above illustration, the cone of light initially illuminates the whole efficient surface. 2q+1 extreme solutions dispersed over this illuminated area are presented to the DM who chooses the most preferred one. The central axis of the cone is then moved to this chosen solution and the cone is reduced to $1/q_{th}$ the cross sectional volume, thereby illuminating a proportionately smaller area around the chosen point. A further 2q+1 efficient extreme point solutions are presented, and the process continues until the DM requires the generation of all efficient extreme point solutions within the reduced light cone. At this stage, the set of efficient extreme point solutions should be of a sufficiently small size to be comprehensible to the DM. Mathematically, this procedure generates a new set of interval bounds at each iteration, with each reduced gradient cone being defined from a new set of critical weights.

As with the ZW method, the DM is required to make holistic comparisons among solutions, and it is assumed that the DM has a linear utility function since only extreme point solutions are ever generated. However there is no guarantee that the method will converge even under the assumption of an ideal DM with a linear utility function [Zionts (1982, pp. 4.21-4.25)].

The fact that only extreme point solutions are generated by this method may have prompted Steuer to develop it further [Steuer and Choo (1983), Greis, Wood and Steuer (1983)]. In this extension a formulation that is different from the weighted sum approach is used. It is the Tchebycheff norm formulation and has the capability of generating every efficient point. A large number of possible weighting vectors are randomly generated and then "filtered" to choose a subset of the most dissimilar vectors, where the filtering process is based on a distance metric. The Tchebycheff formulation is then repeatedly solved using each weighting vector of the filtered set. The set of resulting solutions is again filtered and the DM is presented with a small subset of efficient solutions which will be representative of the efficient set. Again the gradient cone is reduced around the most preferred of these solutions and based on the resulting set of interval bounds a new set of weighting vectors is generated. As the gradient cone shrinks down at each iteration, the sequence of solutions is expected to converge to the most preferred solution; however, given the random component in the method, a rigorous proof of convergence is not possible. This Tchebycheff method is also included in the experiment of Chapter 5.

2.3.2.5 The Method of the Displaced Ideal and the Reference Point Approach

The method of the displaced ideal [Zeleny (1976,1982)] is perhaps more based on empirical studies of decision making behaviour than the methods reviewed thus far. It is based on the concept that choice between alternatives may differ depending on the point of reference which is used. The obvious choice for a point of reference is the ideal solution, which can be displaced as some solutions are excluded from the efficient set. The method, which is interactive, is illustrated in Figure 2.4 below for two objectives.

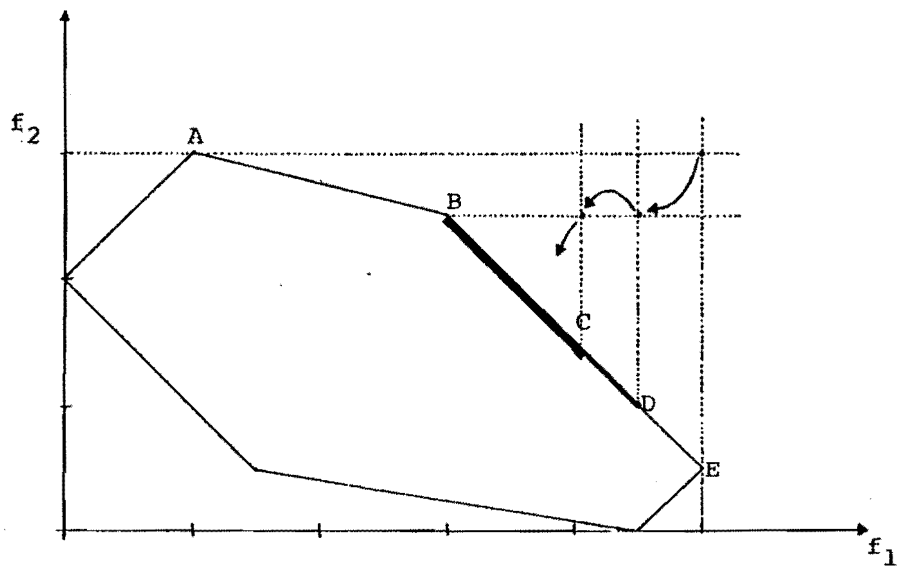

Figure 2.4

The initial compromise set is the entire efficient set ABE which reduces to BD and finally to BC, with the ideal point being displaced each time. The method for determining the compromise set is based on a set of weighted distance metrics with respect to the ideal solution, i.e., $$\left[\sum_{k=1}^{q} w_k(f_k-U_k)^p\right]^{1/p}, \quad 1 \leq p \leq \infty . \qquad [38]$$

Distance metrics are considered in more detail in Section 4.3.

Wierzbicki (1980) makes use of this concept of the displaced ideal in his reference point approach where it is assumed that the DM has certain goals or aspiration levels which are to be attained. The methodology is simple; after being exposed to the extreme solution matrix, the DM specifies a reference point or desired solution. The optimization process is to determine whether or not such a point is in fact attainable, and to present to the DM the efficient point which results from the optimization. If the sequence of reference points, which gives rise to a sequence of attainable points, converges, then the limit is the solution to the MODM. At each attainable point information is given to the DM to aid in the choice of the next reference point. The process of moving from a reference point $\underline{\bar{f}}$ to an attainable point $\underline{\hat{f}}$ is achieved using a scalarizing function $s(\underline{\hat{f}}-\underline{\bar{f}})$. The simplest form of s is that of a distance metric, not unlike that used by Zeleny for determining the compromise set.

However, the distinction between the two methods is that in Zeleny's approach the compromise set is determined, at least in part, by the weights which are placed on each objective. However, in the reference point approach the weights are implicitly incorporated into the reference point $\underline{\bar{f}}$ which is specified by the DM. The reference point approach has come under considerable study by Wierzbicki and his colleagues at IIASA [e.g., Lewandowski and Grauer

(1982)]. This has resulted in a number of practical applications of the method and the production of an interactive package entitled DIDASS which implements the method.

2.3.2.6 Other Methods

While the previous sections do not cover all interactive solution methods, almost all the major concepts have been covered. Two other methods are worth mentioning. Monarchi et al.(1973) propose a method which uses a set of different surrogate objective functions, with the precise form of the surrogate objective being determined by the "nature" of the DM's aspirations. It differs from goal programming in that the goals specified by the DM may be intervals rather than a fixed point. Five possible types of aspirations are catered for. These are upper and lower bounds, equal to, and inside or outside an interval. For each of these aspiration types a single deviation measure is defined and the objective simply becomes the minimization of these deviations. For example, let the aspiration type chosen by the DM be that of inside an interval, i.e., $a_k \leq f_k(\underline{x}) \leq b_k$. The suggested measure of deviation is $d_k = [b_k/(a_k+b_k)][a_k/f_k(\underline{x}) + f_k(\underline{x})/b_k]$. The method involves the use of non linear functions and the iterative procedure is reasonably complex.

A more recent approach advocated by Goicoechea et al. (1979) combines three different solution methodologies. Initially a weighted sum solution is found using equal weights and then the DM's preferences are externalized and incorporated into a utility function. A new set of weights is then found which is consistent with the utility function and from which a second weighted sum solution is determined. This second weighted sum solution should more accurately reflect the DM's preferences. Finally, stochastic elements in the objective function coefficients are then treated by allowing the DM to alter the probabilities of achieving the compromise solution values. Chankong and Haimes (1983b) have modified this method by using the DM's utility function to optimize the MODM, rather than calculating a new set of weights. This method is the only one reviewed which seeks to incorporate possible stochastic elements of the objective functions into a solution method.

2.3.3 A POSTERIORI ARTICULATION OF PREFERENCES

This final group of solution methods to be reviewed, where the DM provides information after solution, were among the first to be developed. The concept of finding all efficient solutions is unrealistic since there are an infinity of them. Consequently, this method of solution has been concerned with the linear form of [1] and the problem of finding all efficient extreme point solutions; i.e., all efficient vertices of the set of feasible decisions X. Thus the general method is simply to generate all efficient extreme point solutions and present them to the DM, who can then choose among them to find the most preferred solution. These methods for generating all efficient extreme point solutions were, at least initially, a theoretical development with little thought given to the role of the DM in the decision making process. While such an approach is likely to require substantial computing time, it does eliminate any methodological subjectivity by simply generating <u>all</u> efficient extreme point solutions, and provided that the DM's most preferred solution is an extreme point, it should be successful. The only disadvantage lies with the size of the set of efficient extreme point solutions which should be large for any MOLP of non-trivial size.

The basic solution strategy for these methods is to begin with an efficient solution. Subproblem tests, such as used in the ZW method [31], are then used to determine adjacent efficient extreme point solutions. A bookkeeping structure is maintained to ensure that all adjacent points to a given extreme point are identified and that each extreme point is visited only once. The process terminates when there are no extreme points which have not been examined. The computational requirements are high for other than small problems.

The methods of Evans and Steuer (1973), Yu and Zeleny (1975) and Ecker and Kouada (1978) are capable of finding all efficient extreme point solutions, with the Ecker and Kouada method simplifying the subproblem tests at each step to only a few pivots. Isermann (1977) and Ecker, Hegner and Kouada (1980) provide methods which are capable of generating every efficient solution. Their methods generate the set of all maximal efficient faces which are defined by convex combinations of their extreme points. Yu and Zeleny (1975) also propose an approach for calculating these efficient faces once the efficient extreme point solutions are known. However this approach is impractical for all but the smallest problems, since it implicitly enumerates all possible combinations of the extreme points, while solving a subproblem for each combination. For example, the small MOLP of Section 4.9 required 21 small LP subproblems to be solved in order to correctly identify the three efficient faces.

It is, however, reasonable to assume that the DM will only be interested in a subset of the set of efficient extreme point solutions. On the basis of this assumption, Ecker and Shoemaker (1981) reduce the size of the efficient set analytically by defining "types" of solutions deemed to be more desirable than others. Morse (1980) and Torn (1980) demonstrate the use of clustering techniques to group the set of efficient extreme point solutions into various types. A dendogram, which is a hierarchical agglomeration from individual solutions into one final solution type using the form of a tree diagram, is used to provide valuable information to the DM on the structure of the efficient set. And in another approach, which also presumes that the set of efficient extreme point solutions has been generated, Levine and Pomerol (1984) demonstrate the use of an interactive method entitled PRIAM. This approach, which is based on artificial intelligence methods, helps the DM explore the efficient set and find a most preferred solution.

Given adequate computing power, these methods would seem to be well suited to reasonably small MOLP problems. It should be noted that in order to find the <u>most</u> preferred solution, it is likely that some searching in the vicinity of the most preferred extreme point solution would be necessary.

2.4 GROUP DECISION MAKING

The previous sections have assumed only a single DM; which in view of real world decision making, especially as it relates to MODM problems, is an unrealistic assumption. Many decision problems, especially in the area of social policy with its associated democratic philosophy, cannot adequately be dealt with under the assumption of a single DM. It is not difficult to appreciate the unsuitability of many of the interactive methods previously discussed with, for example, four or five DM's seated around a computer terminal seeking to provide their collective MRS at a given iteration. It would also be fair to say that there are few, if any, approaches to group decision making which have successfully come to grips with the problems and issues to be examined.

Although the theory of individual decision making has continued to progress, the group situation has proved less tractable. Dalkey (1976, p.46) points out that "attempts to formulate a theory of group decisions have run into a spate of problems that could loosely be called paradoxes of aggregation". Arrow (1951) formalized this aggregation problem with his general impossibility theorem which states that, under an a priori reasonable set of conditions, it is not possible to aggregate individual preferences into a group preference relation. A well known example is that of the voting paradox [e.g., Chadwick (1971, pp.128-129)]. If, however, Arrow's conditions are relaxed to allow interpersonal comparison of utilities, a group preference relation can be derived.

These methods, which allow interpersonal comparison of utility, seek to calculate parameters for standard form models in order to form a group preference relation. The models are quite similar to those of utility theory where the utility function is decomposed to be a function of each objective (e.g., [12] and [13]). Two common standard form models for group utility [Goicoechea et al. (1982, p.344] are given below (for p DM's)

$$\text{additive} \quad : \quad W(\underline{x}) = \sum_{i=1}^{p} k_i V_i(\underline{x})$$

$$\text{multiplicative} \quad : \quad kW(\underline{x}) + 1 = \sum_{i=1}^{p} [kk_i V_i(\underline{x})+1] \qquad [39]$$

where W is the group preference relation and V_i is the preference function for individual i. It has been suggested that such utility aggregation models suffer from the practical difficulties of actually quantifying interpersonal utilities. Also, within a group the members tend to favour decisional equality; i.e., an equal vote, in which case an additive model with unequal parameters k_i , i =1,2,...,p may seem "unfair" [Goicoechea et al. (1982, p.352)].

Theoretical approaches to the multiple DM situation include game theory, an interesting delegation process devised by Bodily (1979) which exploits the property of Markov chains, the compromise approach of Yu (1973) and methods of hierarchical decomposition [Banker and Gupta (1980), Nijkamp and Rietveld (1981)]. A well known practical approach is the Delphi Method developed by Dalkey and Helmer (1963), where the emphasis is on aggregating the opinions of a panel of experts, in order to achieve a consensus of opinion. The method involves systematic and controlled interaction with each DM, accompanied with selective feedback. Each DM is expected to be able to justify his or her adopted position, and, in the light of additional information (feedback), revise that position. Tell (1978) demonstrates the application of a revised Delphi technique to planning in the Swedish Defense Sector. A group of officers were chosen as representatives of the Defense Sector and a neutral supervisor was substituted for feedback. The supervisor questioned each officer about his adopted position, utilizing anonymous information from previous interviews, with the end result being the "opinion of an organization" as aggregated from its representative members.

An alternative and yet equally practical approach has been advocated by Hammond and his colleagues [e.g., Hammond et al.(1975)], where the preferences of each DM are externalized using methods of Social Judgement Theory. Hammond et al.(1977) provide an example of group conflict resolution in the context of an employer-union confrontation, where externalization of individual preferences clearly highlighted the issues at the heart of the disagreement, as distinct from those issues over which there really was no disagreement. This approach will be discussed in some detail in Chapter 3.

Despite the inherent difficulties, group decisions continue to be made. Edwards (1977) provides some insight into the group decision making situation by suggesting that disagreement is generally with respect to degree rather than kind. There is usually agreement as to what kind of values are appropriate (i.e., the objectives), with disagreement as to the level of achievement of the objectives.

This concludes the brief look at group decision making and demonstrates some of the difficulties with this situation as well as some structured approaches for dealing with it.

2.5 DISCUSSION

A number of solution methods for the MODM with a single DM have been presented, along with a brief mention of the multiple DM situation and its inherent difficulties. The various solution methods reviewed all require value judgements (i.e., subjective input) from the DM. Therefore the first observation is concerned with the necessary subjectivity of all MODM solution methods. Value judgements are necessary, and in the majority of solution methods these judgements are progressively articulated until a most preferred solution is found.

As solution methods have been developed there has been a noticeable move away from requiring too much of a DM, both in the quantity and quality of information provided. For example the work of Steuer initially began with methods for generating the entire set of efficient extreme point solutions. This was extended to a method of reducing the size of the efficient set whereby the DM a priori specified bounds on the weights, and then further extended to an interactive scheme with the DM progressively providing a little information at each iteration.

Also, the assumption of an ideal DM who always acts in accord with his or her utility function has been relaxed by using fuzzy sets or a probabilistic approach [Sakawa and Seo (1983) and Stewart (1984)]. This is indicative of a trend toward increasing realism, and is further born out by the example of Zionts and Wallenius (1982) who have made some refinements to their earlier method [Zionts and Wallenius (1976)]. Instead of the DM being required to assess total tradeoffs at each iteration, the DM now has to choose between two adjacent solutions. This is virtually the same information presented in a different manner; which Zionts and Wallenius found from their experience to be better. And with the advent of computer graphics, information can be presented to the DM in a much more comprehensible form [e.g., Winkels (1982) and Ho (1985)]. This, then, has been one of the major trends in the development of MODM solution methods; namely the adoption of a more realistic stance, achieved by reducing the requirements placed on the DM and thereby making his or her participation in the decision making process as easy as is possible.

Consistent with this movement toward greater realism has been an increasing scepticism of the ability of utility theory to meaningfully capture the preferences of the DM. White (1982) has raised some fundamental questions regarding the underlying assumptions of multiple objective interactive programming generally. And in a carefully controlled experiment, de Neufville and McCord (1983) have sought to measure the validity of assessed utility functions. For each subject a utility function was assessed using different methods. They conclude that "methods which should theoretically produce identical functions do not do so; they fail by wide margins, easily greater than 50%" (p.16). This leads into Chapter 3 where behavioural issues of decision making will be considered along with their implication for solution methods.

CHAPTER 3 BEHAVIOURAL ISSUES OF DECISION MAKING AND THEIR IMPLICATIONS FOR MODM SOLUTION METHODS

3.Ø INTRODUCTION

As was briefly mentioned at the conclusion of Chapter 2, methods for solving MODM's have only more recently begun to take account of the actual decision making behaviour of the DM. The purpose of this chapter is to examine empirical evidence relating to human decision making behaviour. The question as to what constitutes rational behaviour will first be addressed, followed by some empirical evidence regarding choice behaviour especially as it contrasts with the axioms of traditional utility theory. This leads into strategies of choice and methods by which such strategies can be captured or modelled. Social Judgement Theory, as an approach to modelling choice behaviour especially in the group situation, will be examined in some detail. The chapter concludes with the implications for MODM solution methods. This is in the spirit of Wallenius (1975, p.1394) who suggests that "a logical direction for future research would be to attempt to better adjust the methods to match the characteristics of a human decision maker...".

3.1 RATIONAL BEHAVIOUR AND OPTIMIZING

Classical economics has defined rational man to be a utility maximizer. This is a normative approach where only the best is good enough. In fact it has been suggested he is so rational that "he would only read in bed if the value of reading exceeded the value (to him) of the loss of sleep suffered by his wife" [Simon (1978, p.2)]. The theory of rational choice under uncertainty is now well developed (from von Neumann and Morgenstern (1947) through to Keeney and Raiffa (1976)), with the sole criterion for rational choice being the maximization of expected utility.

An acceptable definition as to what constitutes rational behaviour is difficult to find. Classically, rationality was defined as the ability of an individual to select means to achieve goals and objectives. Alternatively, rationality can be defined in terms of the goals and objectives which the individual himself adopts. Such a definition is "ends oriented", in contrast to the above "means oriented" definition. Einhorn and Hogarth (1981) give the following example as an explanation of these two definitions. An individual is charged with a serious crime. The prosecution argues that the way in which the crime was planned and executed is evidence of rational behaviour, to which the defense responds by submitting that the very goal to commit such a crime demonstrates irrationality. Zeleny (1976) offers a further definition in his displaced ideal model of decision making. His axiom of choice is "to be as close as possible to the perceived ideal". This definition, like the maximization of expected utility, is means oriented.

While it is difficult to assume behaviour to be rational according to any precise definition, we nevertheless assume that human behaviour does in fact make sense; that even though it may seem anomalous, it is intelligent. As March (1978) points out, we preserve the understanding that behaviour is intelligent even if it is contrary to standard definitions of rationality.

3.1.1 Bounded Rationality

Herbert Simon (1955,1957) has addressed this question concerning the rationality of decision making. He introduced the concept of bounded rationality, founded on the assumption that decision making takes place within a number of constraints. The constraints are concerned with the properties of human beings as problem solvers as they function within an uncertain and complex environment. Simon suggests that humans develop decision procedures which are sensible given these constraints, but do not appear sensible when they are removed. Classical decision theory, in assuming an idealized decision maker whose preferences conform to the axioms of utility theory, effectively removes the constraints. Consequently prescriptions which appear sensible in theory prove to be less so in practice.

So although man may want to optimize within a choice situation, his optimization within the biological constraints on his ability as a problem solver result in a "satisficing" rather than an optimizing strategy. Instead of looking for the sharpest needle in the haystack, satisficing man will stop looking when he has found a needle sharp enough to sew with [March and Simon (1958, p.141)]. However, given that man is intelligent, he will not satisfice when he can just as easily optimize. To extend the haystack example: if the man has at his disposal a sufficiently powerful magnet to quickly find all needles, then he has improved on the initial satisficing solution. The search process, aided by technology, is closer to "optimal".

While Simon has defined "satisficing man", Keen (1977) provides a further definition; that of "apprehensive man". Apprehensive man tends to anticipate adverse events, is always ready to seize an opportunity and tends to learn through apprehension rather than comprehension; i.e., through perception and feeling rather than formal understanding. This view is supported by the empirical evidence of the following sections.

Optimality generally means the best, and is a trademark of the normative model or approach. The optimal decision either maximizes or minimizes some specific criterion but is conditional on environmental assumptions and a specified time horizon. A linear programming model will, for a given objective, optimize to find the single best solution. It is not, as the satisficer would say, "good enough"; it is the best (or perhaps best equal). It is, however, unlikely to be best given a different objective and due to uncertainty, may in the future prove to be far from the best solution. Optimality is strictly a relative rather than an absolute concept. Miller and Starr (1967, p.51) put it well. "It is always questionable whether the optimum procedure is to search for the optimum value...". Man's attempt to optimize in a given situation may well result in a satisficing strategy with the solution being non optimal in the sense of a normative model, but optimal as far as the decision maker himself is concerned.

3.2 EMPIRICAL STUDIES OF DECISION MAKING BEHAVIOUR

The focus of this section is descriptive, highlighting what often seems unexpected choice behaviour in the light of expected utility theory.

3.2.1 Reflection Effect.

Empirical studies have found that the manner in which a task or decision situation is presented to a DM has a significant effect on final choice outcomes. The "reflection effect" [Kahneman and Tversky (1979)] is one compelling example of how preferences of individuals are often reversed when a decision situation is restated negatively. It can be illustrated as follows.

It is expected that a certain flu will kill 600 people this year. You are faced with two options:

A1: save 200 people for sure
B1: save 600 people with probability 1/3
save no people with probability 2/3

The decision situation is then restated negatively as

A2: 400 people will die for sure
B2: none will die with probability 1/3
600 people will die with probability 2/3

In this and other similar experiments they found that the majority of individuals reversed their preferences from A1 to B2. They concluded that people are risk averse in the positive domain and risk taking in the negative domain.

3.2.2 Certainty Effect.

Another departure from the theory of rational choice has been labelled the "certainty effect" [Kahneman and Tversky

(1979)]. This indicates that people exaggerate their preference for outcomes that are considered certain, relative to other probabilistic outcomes. Again Kahneman and Tversky provide some compelling examples, as in the following choice situation.

A1: 50% chance to win a six-week tour of England, France and Italy.

B1: A two-week tour of England for sure.

A2: 5% chance to win a six-week tour of England, France and Italy.

B2: 10% chance to win a two-week tour of England.

At least two thirds of the subjects chose B1 and A2. In contrast, the expected utility model would prescribe B2 if B1 were chosen in the first choice situation.

3.2.3 Biases [Hogarth (1980; Chapter 6)]

Limited memory storage and retrieval operations account for a number of behavioural biases. One is concerned with information retrieval; specifically, the availability of information. For example, are words beginning with "re" more common than words ending with "re"? It is easy to assume that the ease of recall from memory is proportional to the frequency of occurence. In fact, more words end with "re".

A second bias is concerned with two differing sources of information; that concrete information is more salient than abstract information. Abstract information that Motor World rated Volvo as the best Swedish car on the market, supported by an analysis of consumer's garage repair bills, is likely to be less salient in memory than concrete information from your uncle who expounds that his Volvo has cost over $5000 in repairs over the last two years. The concrete information is weighted more heavily than the abstract information. It should be noted, however, that these two biases of availability and salience (concrete / abstract) do not only apply to information. They are also valid for strategies and methods of judgement.

A third bias concerned with the role of memory is hindsight bias, where knowledge of the final outcome can make the prior decision look trivial. A recent discussion in our department serves to demonstrate this:

> "How do you know that the output from this model is correct?"
> "The results are what we expected."
> "Then why bother using the model?", was the reply.

With hindsight, the result often seems more obvious than it did at the point of decision.

3.2.4 Other Behavioural Results

Three other behavioural results relevant to the choice situation will be mentioned. The first concerns the perception of the decision problem. Bruner and Postman (1949) provide a good example where playing cards were presented to subjects who were asked to identify them. Some cards were changed in that while all had the correct insignia, some of the hearts were black. Subjects rarely described the black hearts as such; responses ranged between black and red, and generally a greyish colour was indicated. Expectations had a considerable effect on what was actually perceived.

A second result had been described by Tversky and Kahneman (1974) as "anchored" judgement or judgement by adjustment. They suggest that a natural starting point or anchor is used as a first approximation to the judgement. This anchor, which acts as a point of reference, is then adjusted as additional information is provided. Also, this anchoring of judgement provides the rationale behind Zeleny's (1976,1982) model of the displaced ideal, where he suggests that the ideal solution is the obvious starting point or anchor.

Thirdly, it has been found that increasing the amount of information presented to the DM can prove to be a mixed blessing. Based on experimental work, Jacoby et al.(1974) have suggested that there are two different effects as the amount of information used by the DM increases. First the DM is more confident that the correct choice has been made, and secondly, the quality of his or her predictions are likely to decrease.

3.3 STRATEGIES OF CHOICE

So far some behavioural aspects of choice behaviour have been described. This section will examine strategies or methods of judgement.

There has been a rather clear split as regards research in the general area of decision making; psychology has focussed on the processes of choice and economics on the results of choice [Simon (1978)]. Both are covered in this section. The previous sections are but a sample of the empirical evidence that human choice behaviour is persistently contrary to the prescriptions of expected utility theory. However, as Wallsten (1980) points out, all this evidence suffers from the problem of having little predictive value in that "they have analysed the scratch but not the itch". Modelling efforts have in the main been directed toward describing or "capturing" the judgement policy of a DM. (One exception to this is Prospect Theory, advanced by Kahneman and Tversky (1979)).

3.3.1 Information Processing Strategies

The information processing strategies of a DM have been dichotomized into compensatory and non-compensatory strategies. Compensatory strategies are essentially holistic; all information is evaluated, analysed and traded off to arrive at a decision, whereas in non-compensatory strategies, decisions are based only on selective items of information. Examples of non-compensatory models include:

1. Conjunctive model. A cutoff point is set on each attribute or dimension and all alternatives below that are eliminated.

2. Lexicographic model. The DM chooses that alternative which ranks best on the most preferred dimension. No other dimensions are considered unless there is a tie. If so, the next most preferred dimension is used to break the tie, and

so on.

3. Elimination by Aspects (EBA) [Tversky (1972)]. This is a sequential processing model and operates as follows. At each stage one dimension or aspect is selected and a minimum cutoff point chosen. All alternatives falling below the cutoff point are eliminated. Tversky cites this example from a television commercial.

"'There are more than two dozen companies in the San Francisco area which offer training in computer programming.' The announcer puts some two dozen eggs and one walnut on the table to represent the alternatives, and continues: 'Let us examine the facts. How many of these schools have on-line computer facilities for training?' The announcer removes several eggs. 'How many of these schools have placement services that would help you find a job?' The announcer removes some more eggs. 'How many of these schools are approved for veterans' benefit?' This continues until the walnut alone remains. The announcer cracks the nutshell, which reveals the name of the company and concludes: 'This is all you need to know in a nutshell'." (p.297)

The STEM method [Benayoun et al.(1971)] contains elements of this third model where at each iteration one objective is bounded thereby reducing the set of feasible solutions. Preemptive goal programming also uses a non-compensatory model; i.e., a lexicographic approach.

3.3.2 Linear Models

By comparison, compensatory models have received far more attention in the literature, the most common being the linear model. This model essentially states that

value of an alternative = sum of (relative weight x scale value) of all dimensions

Given its simplicity, the linear model demonstrates remarkable predictive ability, and can reproduce judgements generated by other processes quite accurately. One of the first real indications of this came with the main effect hypothesis of Yntema and Torgerson (1961). Their study indicated that the main effects (i.e., the most important dimensions) dominated the judgement almost to the exclusion of any interactions among the dimensions.

Linear models range from those in which optimal weights are obtained by least squares regression to equal or even random weights. In a review article on linear models, Dawes and Corrigan (1974) cite structural characteristics of the linear model as the main reason for its predictive ability. These include:

1. Independent variables are generally conditionally monotone on the dependent variable.

2. Relative weights in a regression analysis are not affected by error in the dependent variable.

3. Conditionally monotonic functions tend to become more linear in the presence of increasing error in the independent variables, [Lord (1962)].

4. Small errors in the weights (i.e., from optimal) have little effect on the model's predictions. This property is known as robustness. Zeleny (1982) views this property negatively stating that since the actual weights representing the judgement are not unique, it is uncertain what the DM's true preferences among the dimensions are.

Linear models of judgement are a "paramorphic" representation of judgement. That is to say that although the outcome (prediction) of the judgement process can be well simulated by a linear model, such a model is unlikely to reflect the psychological decision processes involved in the judgement. Its acclaim lies with the outcome rather than the process of choice.

3.3.3 Other Compensatory Models

There are many other types of compensatory models. These include the analytic hierarchy approach of Saaty (1977) and information integration theory which focusses on dimensional interactions [Anderson (1971)]. However empirical evidence [e.g. Schoemaker and Waid (1982)] indicates that linear forms generally outperform their non-linear counterparts in terms of predictive ability.

Research has indicated that humans cannot simultaneously integrate and process large amounts of information [e.g., Miller (1956)]. Information is processed in a predominantly sequential manner. Kahneman and Tversky (1979) allude to this with their proposed two phase process involving editing of information in preparation for a second phase of evaluation. Payne (1976) has designed an experiment to examine the pattern of information search in the process of coming to a decision. Subjects were required to choose among a number of apartments which were described on four to eight dimensions such as rent, noise level etc. Verbal reports of the decision process were recorded. Different types of compensatory and non-compensatory strategies were observed to be used, often sequentially. For example, one subject used a non-compensatory strategy not unlike EBA to narrow it down to two alternatives and then traded off the various dimensional "scores" to arrive at a final choice. It is reasonable to expect that the DM will use a number of different judgemental strategies or models, in some sequential fashion, in order to arrive at the final choice outcome.

Any compensatory model can be thought of as a compromise strategy among the model dimensions. A crucial issue in the execution of a DM's compromise strategy is that of judgemental inconsistency. Dudycha and Naylor (1966, p.127) stated that "humans tend to generate correct strategies but then, in turn, fail to use their own strategy with any great consistency". Their suggested approach was to first capture the judgement policy of the DM and then replace him or her by a machine which implements the policy. The origin of such inconsistency is not well understood, however it is likely to be a function of human limitations previously discussed and the uncertain nature of the environment. In focussing on the probablistic nature of the environment, Brunswik (1956) developed the "Lens Model" in an attempt to explain the successes and failures of an organism in an uncertain environment. The environment is uncertain because the cues by which it represents itself are not entirely trustworthy.

Consequently, Brunswik suggests that "the limitations in the dependability of single-cue variables force an uncertainty-geared probablistic strategy on perception" (p.140). Development of the Lens Model along with a regression approach for capturing the judgement policy of a DM, has given rise to a branch of psychology known as Social Judgement Theory (SJT).

3.4 SOCIAL JUDGEMENT THEORY - THE LENS MODEL

The Lens Model can be described as follows;

> " There is a judge who must make decisions on the basis of a set of cues which are only probabilistically related to a criterion."

For example, a doctor (the judge) makes a decision regarding the state of a patient (the criterion) based on his or her observations of a set of symptoms (cues) which are a reflection of the patient's true state.

The Lens Model includes the impact or effect of the environment on the decision making process. Sources of uncertainty in the model are:

1. The validity of the cues with respect to their true state.
2. Inconsistency in the execution of the judgement strategy.
3. The uncertainty in predicting the true state by weighting or combining the cues.

Figure 3.1 on the following page represents the Lens Model.

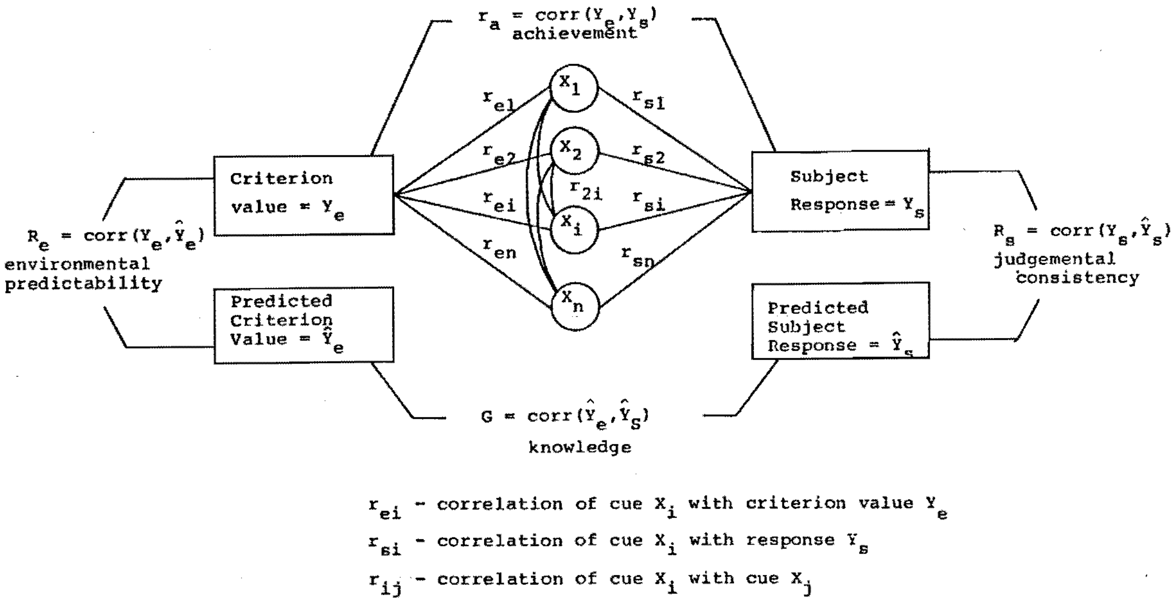

Figure 3.1 The Lens Model

Social judgement theorists [Hursch et al. (1964), Tucker (1964)] have developed the following mathematical representation of the Lens Model. The relationship of the cues to the true and judged states gives rise to two regression equations.

$$\hat{Y}_e = b_{e1}X_1 + b_{e2}X_2 + \ldots + b_{en}X_n$$
$$\hat{Y}_s = b_{s1}X_1 + b_{s2}X_2 + \ldots + b_{sn}X_n$$

The Lens Model equation can be stated in terms of correlations,

$$r_a = GR_eR_s + C[\ (1-R_e^2)(1-R_s^2)\ ]^{1/2} \qquad [1]$$

The parts of this equation have an intuitive interpretation.

r_a - correlation between the true and judged states (Ye and Ys). It is a measure of achievement.

G - correlation between the estimates of the true and judged states ($\hat{Y}_e$ and $\hat{Y}_s$). It is called knowledge and measures how well the DM has perceived the environmental relationships. It is the achievement that the DM would have had if he or she used the Ye regression equation.

R_e - correlation between $\hat{Y}_e$ and Y_e, and is a measure of task validity; i.e., how well the cues reflect the true state. Thus it measures how well the $\hat{Y}_e$ linear equation fits the environmental data.

R_s - correlation between $\hat{Y}_s$ and Y_s and is known as cognitive control or consistency. It is a measure ot the control a DM has over his or her judgements.

If there are no cue interactions, the equation [1] simplifies to

achievement = (knowledge)(task validity)(cognitive control)

$$r_a \quad = \quad GR_eR_s$$

Including interactions, C is the correlation between residuals, $(\hat{Y}_e - Y_e)$ and $(\hat{Y}_s - Y_s)$.

A significant result of this formulation is that G is statistically independent of R_s. Therefore in this framework, inconsistency in the execution of judgements can be separated out from a DM's knowledge (i.e., his or her actual judgement policy). In an essay which examined a number of group interaction studies, Hammond and Brehmer (1973) found that as a result of interaction, subject's cognitive systems converged (G increased) while cognitive control (R_s) decreased. The result was that while there was agreement in principle, the loss of cognitive control produced disagreement in fact. Also, subjects were generally unaware that their actual policies did converge as a result of interaction. These results have considerable implications for group decision making.

Another distinguishing feature of SJT is that feedback is cognitive (i.e., policy oriented) rather than concerned with outcomes only. Focus is on the strategy of choice rather than the results of choice.

This detailed discussion of SJT and the Lens Model has been included here, as the theory will be used in a MODM solution method to be developed in Chapter 6.

3.5 IMPLICATIONS FOR MODM SOLUTION METHODS

This section examines the implications of the previous material on methods for solving MOLP problems.

3.5.1 Utility Theory

Sufficient examples have been cited to question the universal applicability of expected utility theory as either a descriptive or prescriptive theory of choice behaviour. And the practical difficulty of actually assigning utility values has not been addressed either. Some dimensions such as profit, noise or safety measures can be assigned utilities with reasonable confidence. However to place values on dimensions such as dignity or human life is not easy. The view of Keeney and Raiffa (1976, p.25) that "somehow we must learn that our grief should rise monotonically with the magnitude of a catastrophe" is difficult to accept.

However the pioneers of this theory make some useful and pragmatic comments in defending utility theory. Morgenstern (1979) concentrates upon the approximating nature of many theories, and how they are applied despite their limitations; e.g., Newtonian mechanics is not disregarded even though it does not explain the behaviour of light. And Raiffa has commented [Bell et al.(1977, p.435)] that he personally, after much searching, can find no adequate substitute. It seems as if the methods available to the physical sciences are not (as yet) appropriate for measuring human behaviour[1]. Expected utility theory is of much value, especially given the absence of any better theories and will continue to provide a base for comparison. However, the implication for MODM solution methods is that in using a method the DM should not be penalized if his or her behaviour is not consistent with the axioms of the theory.

Rational choice involves two kinds of guesses. The first is the future consequences of our actions and the second is our future preferences for those consequences [March (1978)]. Decisions are by nature anticipatory, characteristically asking, "What will happen if...?" This highlights the time dimension of decision making, looking ahead to the future. Within the framework of a MODM model, the consequences are provided by the model; they are the efficient solutions. The DM's role is to exercise his or her preferences among the solutions and choose. Thus the MODM is no longer strictly

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1. Despite being made some 24 years ago [Churchman (1961, pp.245-246)], this comment is still relevant.

seen as being normative; it is more a vehicle for the generation of future consequences. The optimization is only concerned with discriminating among efficient and inefficient solutions. (It bears repeating at this point that the MOLP model is assumed to be able to generate realistic solutions, or else any solution method will be inappropriate.)

3.5.2 Decision Making as a Learning Process

More pragmatically, Hogarth (1980, p.ix) has made two observations about decision making.

1. People generally are unaware as to how they make decisions or why they prefer one alternative to another.
2. People show little concern for the quality of their own decision making processes.

And Fischoff, Slovic and Lichenstein (1980, p.117) validly ask the question, "What happens, however, in cases where people do not know, or have difficulty appraising, what they want?". In general a DM is unlikely to be sufficiently familiar with a problem (and the model which characterizes it) to be able to prespecify exact preferences. Preferences about future consequences can only be formed as information about the consequences is revealed to the DM. Baum and Carlson (1974) suggest that a preferred decision has to be learned by DM, and this learning is often best achieved by a process of interaction between the model and the DM. Thus as information regarding the decision is progressively revealed, the embryonic preferences grow and take shape.

Any MODM, when a DM first approaches it, represents a potentially ambiguous situation. This is distinct from the situation of uncertainty where the outcomes are known and the uncertainty surrounds the probability of their occurrence. Faced with an ambiguous situation, where there is (at least initially) no clear understanding of the possible outcomes or consequences, the DM is likely to find it difficult to formulate goals. Gimpl (1985) has argued that intolerance of ambiguity is likely to result in the specification of inappropriate goals which must of necessity be based on biased information, since the DM has already approached the problem with prior expectations. And the effect of prior expectations may not be insignificant as, for example, was demonstrated by the experiment of Bruner and Postman (Section 3.2.4). This ambiguity can in part be resolved by providing the DM with information from the extreme solution matrix. This provides a range for each objective and presents to the DM the extremes of the environment within which he or she must operate. (There will, however, also be many other elements of the environment which are not described by the model).

Since ambiguity cannot be eliminated short of presenting the entire efficient set to the DM, a certain amount of responsibility also falls on the DM to "learn" his or her preferences as outcomes are progressively revealed and endeavour to not be conformed by prior expectations. Gimpl suggests that "to properly take advantage of a new situation a series of random move testing experiments is required" (p.11). The interactive method of Steuer and Choo (1983), see also Section 5.2.4, operates in this manner; randomly generated solutions are presented to the DM, who chooses one as being indicative of a direction worthy of further exploration. Sufficient outcomes are presented at each stage for the DM to form preferences as the process continues. In contrast, MODM methods which require a priori specification of preferences, such as goal programming, place a difficult burden on the DM in the face of ambiguity. The concept of targets or goals is valid, however they can only be realistically specified once the DM has become familiar with the set of possible outcomes.

A further important implication is that the learning process consisting of interaction between model and DM may well be of as much value as the final choice outcome, as for example, insights into some tradeoffs become apparent or as the DM is confronted with his or her actual decision making strategy.

Also, the progressive formation of preferences at each iteration in an interactive solution method will be affected by the manner in which information is presented and the quantity of it. (The results of the experiment of Chapter 5 provide some insight into this area and will be discussed in more detail in that chapter). The manner in which the interactive schema is set up, and the type of response required (i.e., task representation), will all have an effect on the DM's choice behaviour. As a practical example of this, a graduate class together attempted to solve a MOLP using the approach of Martinson (Section 4.7.3) where at each iteration it was required to specify a set of weights for the objectives. It was found that as the period of interaction increased in length, the DM's were much more willing to accept a satisficing solution than to continue on to find "better" solutions. Other considerations such as room temperature and other engagements continued to gain greater prominence to the extent of finally dominating their preferences. The current environment has a considerable effect on the judgement process. Brunswik has sought to incorporate the environment into the decision process in his Lens Model. And more recently Bell (1982, p.106) has advocated that "management science must learn to recognize these pressures and allow for them in an analysis. This may require a thorough understanding of the manager's environment and perspective in order to provide a useful decision support system.".

While Phelps and Shanteau (1978) indicate that humans may be able to comprehensively process more information than previously thought possible, there still exists a relatively low ceiling on the number of dimensions which a DM can consider simultaneously. Miller (1956) has suggested about seven dimensions. This refers to as the number which should be able to be dealt with "well" under a compensatory strategy, although in many circumstances seven is likely to be too high. More dimensions could probably be included if non-compensatory strategies were also used.

One implication is that since DM's have been observed to use both compensatory and non compensatory decision strategies, MODM solution methods should therefore make provision for both. Furthermore, the fact that these decision strategies are predominantly sequential in nature, would tend to favour sequential or interactive methods of solution. Consideration may also need to be given to a priori reduction of the number of objectives that need to be considered by the DM. Chapter 7 discusses several possible analytical approaches for this.

3.6 CONCLUSIONS

Although human choice behaviour may not always appear to be rational, it is accepted that choice behaviour is intelligent as it is carried out within the constraints of human limitations and environmental uncertainty. The result is limited or bounded rationality which, as March (1978, p.598) states, " is not necessarily a fault in human choice to be corrected, but often a form of intelligence to be refined by the technology of choice rather than ignored by it". Human decision making is limited rather than flawed [Hammond (1976)]. This is an important distinction, for were it flawed, this would imply the need for correction, and prescription would therefore be appropriate. However, if we accept that it is limited, then it requires support or aid. Hence the concept of a decision aid or decision support systems.

Even if a less arrogant stance is taken; namely that human decision making is indeed flawed, the concept of aiding the DM is still appropriate. This is because it is unlikely that there would be any agreement on as to what constitutes the "correct" prescription for decision making behaviour. Consequently we return to the approach of supporting the DM, if for no other reason than the fact that decisions will continue to be made, with or without support.

The implication for MOLP solution methods is that the DM should be supported in his or her search for the most preferred solution, rather than be required to conform to certain (often only implied) conditions for the method to perform as it should, (e.g., Oppenheimer (1978), in his proxy tradeoff method, requires the DM to "redo" her choice if it is found to be inconsistent). This view is not new. Soland (1979) suggests that for an interactive process to be useful it

- must be accepted by the DM
- should be relatively easy to use
- should leave the DM in control.

There needs to be an appropriate balance between analysis, as provided by the MODM solution method, and intuition on the part of the DM [McGinnis (1984)].

The limitations of humans as problem solvers need to be accommodated; the DM being allowed sufficient freedom to move towards a preferred solution, with the method providing appropriate support in the process.

This analysis clearly shows that a number of interactive solution methods may well be inappropriate when examined in the light of actual choice behaviour. It seems, however, that currently a pragmatic stance is taken; that if a particular method performs "well", then it should be used regardless of the underlying assumptions. French (1984) sounds a note of caution as to just how good a measure of performance is the fact that the DM is satisfied with a particular solution. He states that there should be sound methodological grounds for any method that is used.

It has not been the intention of this analysis to indict. The intention has instead been to observe with the aim of gaining a better understanding of decision making behaviour, so that MODM solutions methods to be developed will be relevant and appropriate. Also, demonstration of possible weaknesses in individual MODM solution methods should result in a wiser and more appropriate use of them.

CHAPTER 4 PROPERTIES OF MODM'S AND SOLUTION METHODS

4.Ø INTRODUCTION

In this chapter some of the terminology and definitions of Chapter 2 are developed further, with an emphasis on a certain characterization of an efficient solution which will be called the maxmin or P1 formulation. The motivation for this is twofold; firstly to develop a better understanding of MOLP's and specifically the maxmin formulation, and secondly to provide a framework or basis for some new solution methods.

After some introductory terminology, the theory of the maxmin formulation is developed. From this two possible solution methods are discussed. A small example is used to illustrate both the theory and the solution methods developed. Included in this final section is a discussion of the e-constraint formulation and it's similarity with the maxmin formulation.

4.1 DEFINITIONS

The MODM will be restated.

$$\begin{aligned} \text{'MAX'} \quad F(\underline{x}) &= [f_1(\underline{x}), f_2(\underline{x}), \ldots, f_q(\underline{x})] \\ \text{s.t.} \quad & \underline{x} \in X \end{aligned} \qquad [1]$$

where X is the set of feasible decisions,

$$X = \{\underline{x} \in R^n : g_j(\underline{x}) \leq \emptyset \ , \ j=1,2,\ldots n+m\}$$

and $F(\underline{x})$ is a vector of scalar valued objective functions defined on $\underline{x}$. It will be assumed that the set X is convex.

The linear form of [1] can be stated as

$$\begin{aligned} X = \{\underline{x} \in R^n : & \sum_{j=1}^{n} a_{ij}x_j \le b_i \quad , \; i = 1,2,\ldots,m \\ & x_j \ge \emptyset \quad , \; j = 1,2,\ldots,n\} \\ f_k(\underline{x}) = & \sum_{j=1}^{n} c_{kj}x_j \quad , \; k = 1,2,\ldots,q \end{aligned} \qquad [2]$$

The following definitions will also be used.

Let

$Z \in R$ be the set of feasible decisions in objective space

$N \in Z$ be the set of efficient solutions in objective space

$E \in X$ be the set of all efficient points in decision space

4.2 MAXIMA AND MINIMA OF OBJECTIVES

The extreme solution matrix, as defined in Section 2.1.3, contains a considerable amount of useful information about any MODM, and provides a good approximation to the range of efficient solution values for each objective. The maximum and minimum values of each objective are

$$\text{maximum} = U_k = \max_{\underline{x}\in X} f_k(\underline{x}) \quad , \quad k = 1,2,\ldots,q$$

$$\text{minimum} = M_k = \min_{\underline{x}\in E} f_k(\underline{x}) \quad , \quad k = 1,2,\ldots,q. \qquad [3]$$

While the U_k values can be found directly from the extreme solution matrix, the true M_k values can only be approximated by the minimum value in any column j , j=1,2,...,q , of the extreme solution matrix.

i.e., $$M_k \simeq \min_{1 \leq j \leq q} f_k(\underline{x}_j^*) , \quad k = 1,2,\ldots,q \qquad [4]$$

where $\underline{x}_j^*$ is the optimal solution when f_j is maximized.

The true value of M_k will always be less than or equal to this approximation, a distinction which is not often made in the literature. (For a recent exception to this, see Steuer (1984, p.135)). Instead, it is generally assumed that the true M_k values are readily available. The true minimum of objective k can only be found once all efficient extreme point solutions have been calculated, a task which will have a prohibitively high computational cost for any reasonably large problem. However, experience has indicated that the approximation of M_k obtained from the extreme solution matrix is usually quite close to the true value. Some of the implications of this difficulty will be mentioned in sections 4.3 and 4.7.8.1.

4.2.1 Inefficient Solutions in the Extreme Solution Matrix

It is possible that some of the solutions which comprise P may be inefficient. Consider Figure 4.1 on the following page which is a possible representation of the feasible set for two linear objectives f_1 and f_2 in objective space.

Solving the problem
$$\begin{aligned} &\text{Max} \quad f_2(\underline{x}) \\ &\text{s.t.} \quad \underline{x} \in X \end{aligned} \qquad [5]$$

will find the correct value for f_2 of 10, however the simplex method (depending on the approach route) may stop at point A which, while being an optimal solution to [5], is inefficient. This generation of an inefficient point occurs when the slope of the maximand of [5] is equal to a portion of the efficient set.

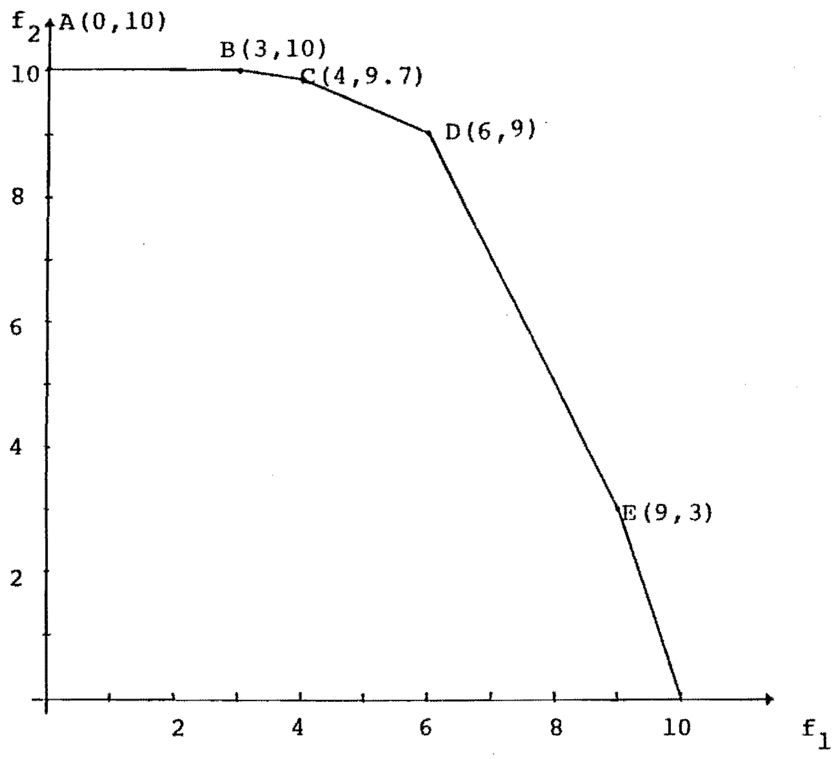

Figure 4.1

This situation can be resolved by choosing an augmented maximand for [5], i.e.,

$$\text{Max} \quad f_k(\underline{x}) + \alpha \sum_{j=1}^{q} f_j(\underline{x}) \qquad [6]$$

where α is sufficiently small not to "tilt" the augmented maximand to such an extent that C is chosen as the optimal solution. An upper bound on α can be established as follows, using the approach of Steuer and Choo (1983).

Let A_k be the set of solutions such that f_k is maximized.

(A_2 is therefore the points along AB.) Consider two solutions

$$\underline{f}^r = (f_1^r, f_2^r, \ldots, f_q^r) \qquad \underline{f}^s = (f_1^s, f_2^s, \ldots, f_q^s)$$

where $\underline{f}^r \in \{A_k\}$ and $\underline{f}^s \in Z$ such that $f_k^r > f_k^s$ for any k $\{1,2,\ldots q\}$. Then α is chosen such that

$$f_k^r + \alpha \sum_{j=1}^{q} f_j^r > f_k^s + \alpha \sum_{j=1}^{q} f_j^s$$

$$\rightarrow \alpha < (f_k^r - f_k^s) \Big/ \left(\sum_{j=1}^{q} (f_j^s - f_j^r) \right)$$

$$\text{for} \quad \sum_{j=1}^{q} (f_j^s - f_j^r) > 0 \qquad [7]$$

The upper bound on α is given as

$$\alpha < \min_{\underline{f}^r \in \{A_k\}} \left\{ \min_{\underline{f}^s \in N - \underline{f}^r} (f_k^r - f_k^s) \Big/ \left(\sum_{j=1}^{q} (f_j^s - f_j^r) \right) : \sum_{j=1}^{q} (f_j^s - f_j^r) > 0 \right\} \qquad [8]$$

This can be illustrated from Figure 4.1. Let $\underline{f}^r = (0,10)$ which is the smallest value of f_1 for $f_2 \in \{A2\}$ where

$A2 = \{ \underline{f} : f_2 = 10 , 0 \leq f_1 \leq 3 \}$.

Then

$$\alpha < \min_{\substack{3 < f_1 < 10 \\ 0 \leq f_2 \leq 10}} \left\{ \frac{10 - f_1}{(f_1 - 0) + (f_2 - 10)} \right\}$$

$$= \min \left\{ \frac{10 - 9.7}{13.7 - 10}, \frac{10 - 9}{14 - 10}, \frac{10 - 5}{11 - 10} \right\}$$

$$= 0.0811 \quad \text{(considering only points C, D and E)}$$

Therefore using an augmented maximand of

$$f_2(\underline{x}) + 0.08 \, [f_1(\underline{x}) + f_2(\underline{x})]$$

will result in solution B.

Since a value for α cannot be derived analytically, the common approach has been to set it equal zero. Alternatively, setting too large a value for α will give a solution where the true maximal value for the objective is not achieved, e.g., solution C in Figure 4.1. In practice, it has been found that when calculating the extreme solution matrix, a useful value for α is of the order of 10^{-6}.

4.3 DISTANCE METRICS AND NORMALIZATION

Many approaches for finding efficient solutions to the MODM [1] incorporate (either explicitly or implicitly) the concept of a distance measure. The commonly used weighted sum approach ([5], Chapter 2) effectively finds a solution where the sum of the weighted deviations (or distances) from the worst solution $\underline{M} = (M_1, M_2, \ldots, M_q)$ are maximized. It is, however, more common for methods which explicitly use a distance metric to be based on the concept of distance from the ideal solution $\underline{U} = (U_1, U_2, \ldots, U_q)$, e.g., Steuer and Choo (1983), Zeleny (1982, pp.130-179) and de Kluyver and Martinson (1979). Or in Wierzbicki's (1980) reference point approach, a distance metric is defined relative to the reference point specified by the DM.

The distance metric, which can be either achievement or deviation oriented, is often first normalized in order to provide a commensurable measure over all objectives. Two common normalizations are (as measures of achievement)

$$d_k^F(\underline{x}) = f_k(\underline{x}) \;/\; U_k \;, \quad U_k > 0 \qquad \underline{d}^F \in (-\infty, 1] \qquad [9]$$

$$d_k^R(\underline{x}) = (f_k(\underline{x}) - M_k) \;/\; (U_k - M_k) \qquad \underline{d}^R \in [0\;, 1] \quad . \qquad [10]$$

These can be interpreted as measures of percentage achievement, which is a very useful measure for comparing the values of different objectives in a solution. The range norm [10] is, in general, more meaningful than the fractional achievement norm [9]. However, as previously mentioned, [10] does suffer from the difficulty of exactly determining the true value of M_k. Consequently, an estimate of M_k $(\hat{M}_k > M_k)$ derived from the extreme solution matrix P, will result in $d_k^R \in [e,1]$ for $0 < e \ll 1$. Also, there is a conceptual difficulty associated with the fractional achievement norm in that it can often give meaningless results. This is illustrated in Appendix 1.

The family of distance metrics used to aggregate the normalized d_k measures of distance are

$$L_p(\underline{x}) = \left[\sum_{k=1}^{q} w_k(d_k(\underline{x}))^p\right]^{1/p} \qquad 1 \leq p < \infty \qquad [11]$$

where w_k is a weight assigned to each distance measure. By defining a single deviational variable $y_k = 1-d_k(\underline{x})$ for each objective, a useful characterization of an efficient solution can be written as

$$\begin{aligned} &\text{Min } \left[\sum_{k=1}^{q} w_k(y_k)^p\right]^{1/p} && 1 \leq p < \infty \\ &\text{s.t. } d_k(\underline{x}) + y_k = 1 && k = 1,2,\ldots,q \qquad [12] \\ &\qquad \underline{x} \in X . \end{aligned}$$

The two special linear cases of [12] will now be examined in more detail, i.e., p = 1 and $p = \infty$.

4.4 THE MINSUM FORMULATION

Letting p = 1 results in a minsum formulation where the sum of the weighted deviations from the ideal solution are minimized. This is equivalent to a goal programming formulation with additive weights, where targets are represented by the ideal solution and the deviational variables only measure underachievement. The formulation is

$$\begin{aligned} \text{Min} \quad & \sum_{k=1}^{q} y_k \\ \text{s.t.} \quad & y_k = w_k(1-d_k(\underline{x})) \quad , \quad k = 1,2,\ldots,q \\ & \underline{x} \in X \\ & w_k \geq 0 \quad , \quad k = 1,2,\ldots,q \end{aligned} \qquad [13]$$

From Soland's characterization of an efficient solution ([4], Chapter 2), any optimal solution to [14] is efficient and, in the linear case, is also extreme. Consequently in the linear case, there exists an infinitely large set of weighting vectors all of which will generate the same optimal solution $\underline{d}^* = (d_1^*, d_2^*, \ldots, d_q^*)$. (Ho (1981) exploits this property in his interactive solution method HOPE). A change in the weighting vector, then, will not necessarily result in a change in the solution. In general, continuous changes in the weights will give discrete changes in the solution values as the solution jumps from one extreme point to another.[1]

There exists a "sister" formulation to [13] known as the maxsum formulation where the weighted sum of the achievements of each objective are maximized. The formulation is given on the following page.

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1. As De Kluyver and Martinson (1979) point out, this may be contrary to the intuition of the DM.

$$\text{Max} \quad \sum_{k=1}^{q} y_k$$
$$\text{s.t.} \quad y_k = w_k d_k(\underline{x}) \quad , \quad k = 1,2,\ldots,q$$
$$\underline{x} \in X \qquad [14]$$
$$w_k \geq \emptyset \quad , \quad k = 1,2,\ldots,q$$

For any given set of weights $\underline{w} = (w_1, w_2, \ldots, w_q)$, the optimal solutions to [13] and [14] are identical (except for the values of y_k).

4.5 THE MINMAX OR TCHEBYCHEFF FORMULATION

In this formulation $p = \infty$. This causes the largest deviation to completely dominate the solution. The formulation is

$$\text{Min} \left\{ \max_{1 \leq k \leq q} w_k(1-d_k(\underline{x})) \right\} \quad , \quad k = 1,2,\ldots,q$$
$$\text{s.t.} \quad \underline{x} \in X \qquad [15]$$
$$w_k \geq \emptyset \quad , \quad k = 1,2,\ldots,q$$

Setting y equal to the largest deviation the formulation becomes

$$P2(\underline{w}) : \quad \text{Min} \quad y$$
$$\text{s.t.} \quad y \geq w_k(1-d_k(\underline{x})) \quad , \quad k = 1,2,\ldots q$$
$$\underline{x} \in X \qquad [16]$$
$$w_k \geq \emptyset \quad , \quad k = 1,2,\ldots,q$$

Again there is a "sister" formulation to [16]; namely a maxmin formulation which seeks to maximize the minimum achievement of each objective. It can be stated as

$$\begin{aligned} P1(\underline{w}) : \quad & \text{Max} \quad y \\ & \text{s.t. } y \leq d_k(\underline{x})/w_k \qquad , \quad k = 1,2,\ldots q \\ & \quad \underline{x} \in X \qquad\qquad [17] \\ & \quad w_k <> \emptyset \end{aligned}$$

The theory of the minmax formulation (P2) is covered in some detail by Bowman (1975), Choo and Atkins (1980) and Steuer and Choo (1983). The maxmin formulation has also been dealt with in the literature, initially by Kaplan (1974) and also by Gupta and Arora (1977) and Posner and Wu (1981). The efforts of the latter group have been directed toward methods for deriving an analytic solution to the maxmin problem, without recourse to linear programming. This is likely to be motivated, at least in part, by Posner and Wu's observation that the linear programming formulation [17] is very degenerate, especially in the early iterations.

4.6 A COMPARISON OF FORMULATIONS

These four different formulations are shown in Figure 4.2 on the following page for a given set of weights $\underline{w} = (w_1,w_2)$ (in two dimensional objective space).

The minsum (and maxsum) solution is the extreme point solution E, with the normal to the objective having a slope of $(w_1,w_2) = w_2/w_1$. The minmax solution is D, being the intersection of the efficient set and a ray from the ideal solution I with slope $(1/w_1,1/w_2) = w_1/w_2$, and the maxmin solution is C. The means by which the maxmin solution is found can be illustrated by imagining a laser situated at the origin whose beam can swing to illuminate any point on the efficient surface. The direction of the beam is determined solely by the weights (w_1, w_2). Kaplan (1974) illustrates this in decision space where the optimal maxmin solution is at the intersection of the ray and the feasible set.

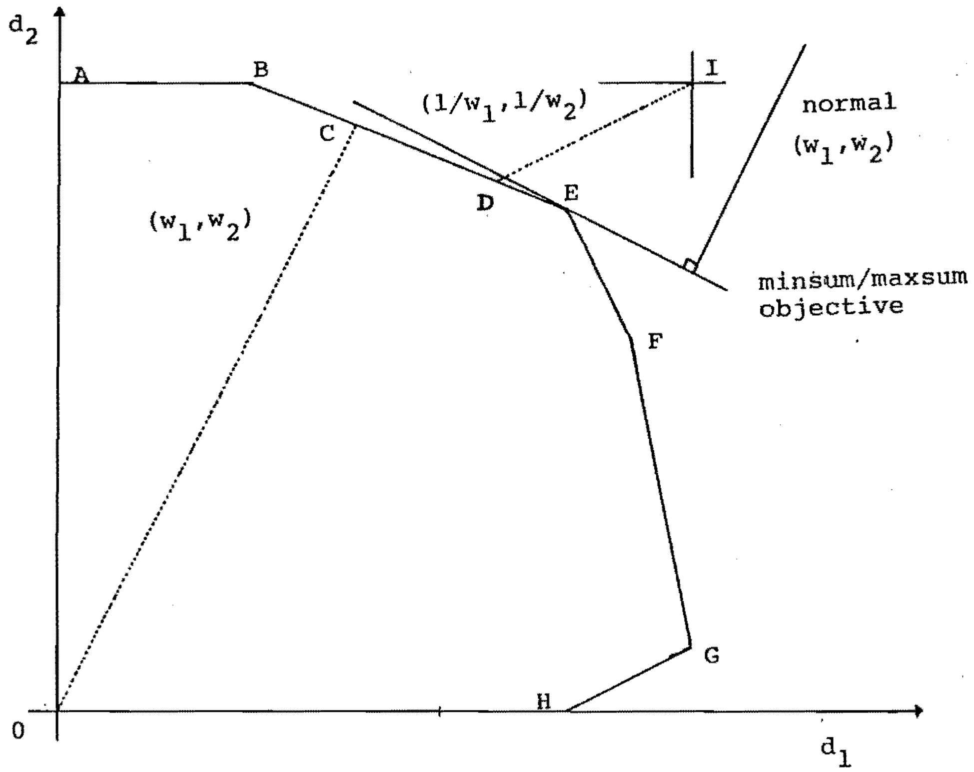


Figure 4.2

Unlike the minsum and maxsum formulations, P1 and P2 give continuous changes in solution for continuous weight changes. This is a consequence of their ability to generate every efficient solution and, unfortunately, some inefficient solutions.

4.6.1 Comparison Between P2 and Maxsum Formulation

This comparison is concerned with the preference structure of the DM which is implied by the P2 and maxsum formulations. The result is obvious for the maxsum case. The weighted sum form of the composite objective function simply implies that the DM has a linear utility function.

However, for the P2 formulation, a simple analysis will be performed in two dimensions. Consider Figure 4.2 where the optimal P2 solution is D for a set of weights (v_1, v_2). Minimizing the following quadratic function, subject to the constraint of line segment BE, will also give D as the optimal solution.

$$v_1(1-d_1)^2 + (v_2/\beta)\,(1-d_2)^2$$

where β is the slope of line segment BE.

This quadratic curve can be likened to a concave utility function of the two objectives d_1 and d_2. Therefore, this would suggest that the implied preference structure of the DM is that of a "sum-of-powers" utility function which has the general form

$$\sum_{k=1}^{q} a_k(S_k+f_k)^{\alpha_k}$$

where $S_k+f_k > \emptyset$, $k = 1,2,\ldots,q$ [see Sakawa (1982, p.391)].

Also, this small example is instructive in that it highlights the impact that the geometry of the feasible set has in determining the solution. It is not only the weights specified by the DM which determine the resulting solution. Instead, the quadratic utility curve is a function of <u>both</u> the weights (v_1, v_2) and the constraint set, as described by β.

Consequently, the extent to which a solution to the MODM reflects the preferences (i.e., weights) of the DM will largely be determined by the shape of the efficient set.

4.6.2 Comparison Between P1 and P2 Formulations

Except for the solution of equal weights ($w_1 = w_2$), P1 and P2 will always give different solutions. P2 is the more conservative formulation, i.e., P1 always gives more of the preferred objective (in terms of relative weights) than P2. This property can be demonstrated as follows (for two objectives).

Consider a section of the efficient set such as BE in Figure 4.2 which has the general equation $\beta d_1 + d_2 = \delta$. It will be assumed that $\underline{d} \in [\emptyset, 1]$ which implies that $\delta \geq 1$ and $\delta > \beta$. Let α be the ratio of the weights, $\alpha = w_1/w_2$. Then considering only the d_1 values

$$\text{For P1 },\ d_1^1 = \delta\alpha/(1+\beta\alpha)$$
$$\text{For P2 },\ d_1^2 = (\alpha+\delta-1)/(\alpha+\beta)$$

P2 is more conservative than P1 if $d_1^1 \leq d_1^2$ i.e.,

$$\delta\alpha/(1+\beta\alpha) \leq (\alpha+\delta-1)/(\alpha+\beta)$$

After some manipulation this reduces to

$$\alpha \leq 1$$
$$\rightarrow \qquad w_1 \leq w_2$$

i.e., d_2 is preferred to d_1. This means that if d_2 is preferred to d_1, then $d_1^1 \leq d_1^2$ and therefore $d_2^1 \geq d_2^2$, i.e., P1 gives more of the preferred objective.

4.7 THE P1 FORMULATION

4.7.0 Introduction

The P1 formulation seeks to enforce a solution such that the weighted achievement of every objective is equal and maximized. This is in contrast to P2 which searches for a solution where the weighted deviations from the ideal solution are equal and minimized.

Consider the Lagrangian of P1

$$L\{P1(\underline{w})\} = y + \sum_{k=1}^{q} \pi_k (d_k(\underline{x}) - w_k y) + \sum_{j=1}^{m} u_j g_j(\underline{x}) \quad . \qquad [18]$$

Let $\underline{x}^*$ be the optimal solution to $P1(\underline{w}^*)$ with solution values $(d_1^*, d_2^*, \ldots, d_q^*, y^*)$. Assume that $\underline{x}^*$ satisfies the Kuhn Tucker efficiency conditions and that each $\pi_k < \emptyset$, $k = 1,2,\ldots,q$. Then

$$L\{P1(\underline{w}^*)\} = y^* \qquad [19]$$
$$= d_1^*/w_1^* = d_2^*/w_2^* = \ldots = d_q^*/w_q^*$$

However, this equal weighted achievement is not always achievable. The more general case is when some $\pi_j = \emptyset$, $j \in \{1,2,\ldots,q\}$. What this in effect means is that although y was maximized at a value $\hat{y}$, $\hat{d}_j$ could not be made small enough to be equal to $\hat{w}_j\hat{y}$. $\hat{w}_j$ was so small relative to the other weights that it ultimately played no part in determining the optimal solution, since the constraint $\hat{y} \leq \hat{d}_j(\underline{x})/\hat{w}_j$ was slack. Conceptually, this result occurs when the ray defined by the weighting vector $\underline{w}$ does not intersect the efficient set. For example, a ray from the origin which intersects the line segment GH in Figure 4.2 will give G as the optimal solution.

It is an important feature of both this and the P2 formulation that only the <u>relative</u> weights are important. Consider modifying the weights by a scalar $\beta > 0$, $\hat{\underline{w}} = \beta \underline{w}$.

$$\text{P1}(\hat{\underline{w}}): \quad \text{Max } y$$
$$\text{s.t.} \quad \beta y \geq d_k(\underline{x})/w_k \quad , \; k = 1,2,\ldots,q \qquad [20]$$
$$\underline{x} \in X$$

Let $\hat{y} = \beta y$, then [20] can be written as

$$\text{Max } \hat{y}/\beta$$
$$\text{s.t.} \quad \hat{y} \geq d_k(\underline{x})/w_k \quad , \; k = 1,2,\ldots,q \qquad [21]$$
$$\underline{x} \in X$$

[21] will give an identical solution to [20] except that the value of the objective function will be proportionately reduced by the scalar β . A consequence of this property is that, at most, only q-1 weights are required to solve P1($\underline{w}$). For example, d_1 can be chosen as a reference objective and all weights calculated as $\underline{w} = (1, w_2/w_1, w_3/w_1, \ldots, w_q/w_1)$.

Furthermore, assume that at a given efficient solution $\underline{x}^*$ to P1($\underline{w}^*$) that only the first p objective constraints are binding, i.e.,

$$\pi_k < 0 \quad , \quad k = 1,2,\ldots,p \qquad [22]$$
$$\pi_l = 0 \quad , \quad l = p+1,p+2,\ldots,q$$

Given this, then at most p-1 weights will be required to determine the solution $\underline{x}^*$. However, since it is not known which of these objective constraints will be binding prior to solution, the result has little practical use.

4.7.1 Efficient Solutions

The P1 formulation may generate inefficient solutions in the situation where a pairwise tradeoff is zero. For example P1 would generate all solutions along line segment AB in Figure 4.2, since the pairwise tradeoff $t_{21} = \delta f_2/\delta f_1 = \emptyset$. In this case the Lagrange multiplier corresponding to the binding objective constraint is identically zero (dual degeneracy). Before this problem of inefficient solutions is addressed, the following theorem concerning P1($\underline{w}$) taken from Lightner and Director (1981) can be stated.

Theorem

A solution $\underline{d}^*$ to P1($\underline{w}^*$) is efficient if and only if there exists a set of weights $\underline{w}^* \geq \emptyset$ and $\underline{w}^* <> \emptyset$ which uniquely maximizes P1($\underline{w}^*$).

Proof

$\longleftarrow$ For $\underline{w}^* \geq \emptyset$ assume that the resulting solution to P1($\underline{w}^*$) is ($\underline{d}^*$,y*) and that $\underline{d}^*$ is inefficient. Then there exists some $\hat{\underline{d}} \in Z$ such that

$$\hat{d}_k \geq d_k^* \quad \text{for all } k$$
$$\hat{d}_j > d_j^* \quad \text{for some } j$$

Thus $w_k^* \hat{d}_k \geq w_k^* d_k^*$ for all k which implies that $\hat{y} \geq y^*$, which is a contradiction since y* uniquely maximizes P1($\underline{w}^*$).

$\longrightarrow$ Let $\underline{d}^* \in Z$ be efficient and define $\underline{w}^* = (d_1^*, d_2^*, \ldots, d_q^*)$. Consider another solution $\hat{\underline{d}} \in Z$, $\hat{\underline{d}} <> \underline{d}^*$, where $\hat{y} = \text{Max}\{\ \text{Min}(\hat{d}_1/w_1^*, \hat{d}_2/w_2^*, \ldots, \hat{d}_q/w_q^*)\ \}$. Then $\hat{y} < y^*$ since $\underline{d}^*$ uniquely maximizes Pl($\underline{w}^*$). If this were not true then

$$y^* \leq \hat{y}$$

$$\rightarrow \quad \underline{d}^* \leq \hat{\underline{d}}$$

Since $\hat{\underline{d}} <> \underline{d}^*$ then

$$\hat{d}_k > d_k^* \qquad \text{for some } k$$

and therefore $\underline{d}^*$ is not efficient which is again a contradiction. This concludes the proof.

4.7.2 The Weighting Vector

This second part of the proof ($\longrightarrow$) confirms the relationship between the weights used and the solution value derived. Let $\underline{d}^*$ be the optimal solution to Pl for a weighting vector $\underline{w}$. Then the solution to Pl($\underline{w}^*$) is also $\underline{d}^*$ if $\underline{w}^* = \underline{d}^*$. The Pl formulation will seek a solution whose values are in the same proportion as the weights.

This result, in incorporating solution values as weights, provides a useful transition between weights and solutions for the MOLP problem. This enables the DM to specify a desired solution rather than a desired set of weights; an approach which should be conceptually easier.

Thus Pl will, for a given desired solution, find an efficient solution "close" to that desired solution while attempting to preserve the relative amounts of each objective. Figure 4.3 below illustrates this. If the desired solution is J, the achieved solution will be H with $y^* < 1$ indicating that the desired solution was infeasible. And conversely, if the desired solution is G, the achieved solution will again be H with $y^* > 1$ which is indicative of a feasible guess that can be improved. The value of y^* carries useful information as to the position of the desired solution relative to the achieved solution.

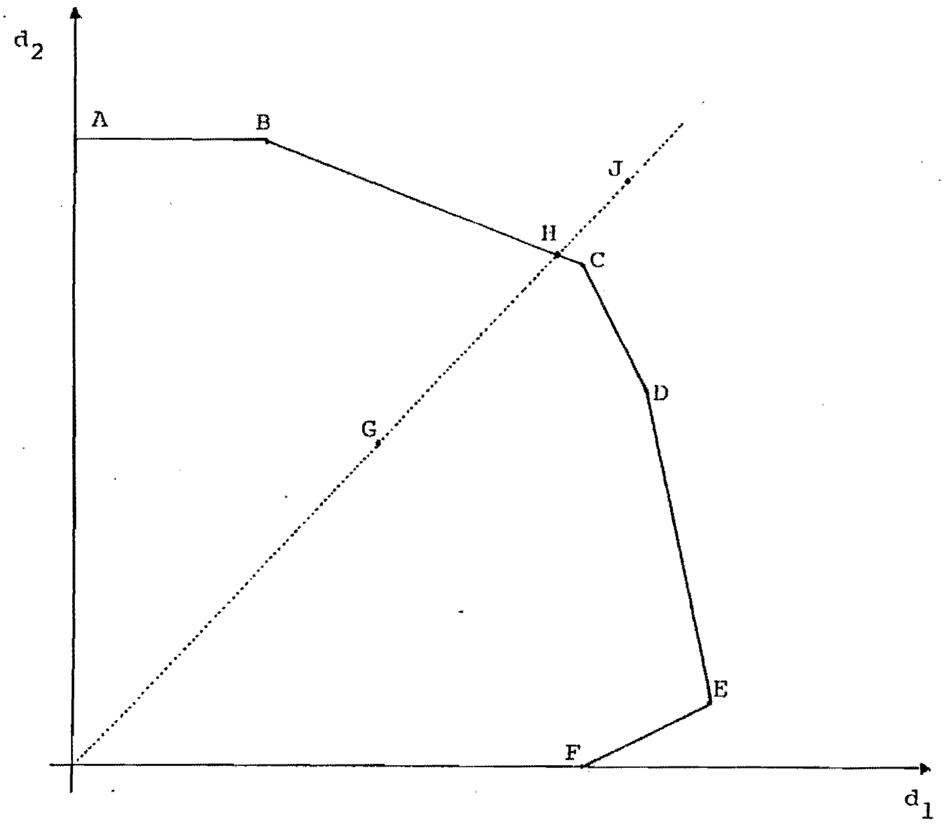


Figure 4.3

4.7.3 A Naive Solution Method

The previous section provides a basis for the following naive solution method for any MODM. P1 is formulated as follows

$$\text{P1}(\underline{w}) : \quad \text{Max } y$$

$$f_k(\underline{x}) - w_k y \geq \emptyset \quad , \; k = 1,2,\ldots,q \qquad [23]$$

$$\underline{x} \in X$$

where $f_1(\underline{x}), f_2(\underline{x}), \ldots, f_q(\underline{x})$ are the original objective functions with no normalization.

The DM chooses a desired solution which is then used as the weighting vector $\underline{w} = (w_1, w_2, \ldots, w_q)$. The achieved solution is presented to the DM who, based on this additional information, chooses another desired solution. The process terminates when the DM is satisfied with the achieved solution.

Obviously this method puts no restrictions on the DM's decision making behaviour, allowing him or her complete freedom at each iteration to move anywhere over the efficient surface. Possible drawbacks of the method may result from it's simplicity, especially the lack of information at each iteration, and the seeming lack of purpose of the method in that it does not propose to move toward or "zero in" on the most preferred solution. The method is simply an aid to help the DM to get close to a good solution.

This method, as a naive solution method, does have merit when compared to two other naive methods documented in the literature. In the naive approach used by Wallenius (1975), the DM chooses a desired solution and is told only whether or not it is feasible; no attempt is made to find an efficient solution. And Martinson (1977) has used a minmax formulation where the objectives were normalized using the fractional achievement norm. In his solution method the DM was required to provide a set of weights which reflected the relative importance of each objective. These were then used to find the achieved solution. Some practical experience with this approach has indicated that the DM often has difficulty in relating the achieved solution to the particular set of weights chosen.

This naive method is one of four methods which will be used in the experiment described in Chapter 5.

4.7.4 Inefficient Solutions

Inefficient solutions are generated, when for a given weighting vector, $Pl(\underline{w})$ is not uniquely maximized, i.e., there exist alternate optimal solutions. The method of dealing with this has been covered by Steuer and Choo (1983) and is similar to that used when generating the extreme solution matrix; namely to augment the maximand. This has the effect of modifying the isoquant as shown in Figure 4.4 below.

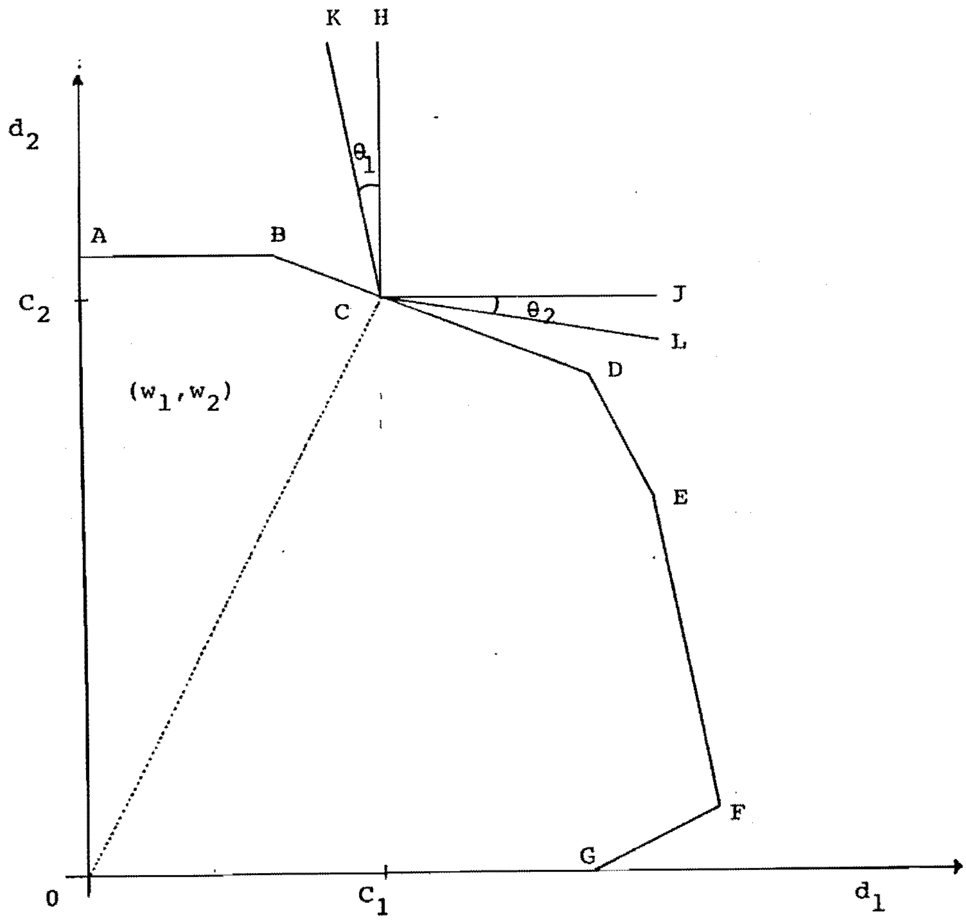


Figure 4.4

The augmented maximand is

$$\text{Max} \quad y + \alpha \sum_{k=1}^{q} f_k(\underline{x}) \qquad [24]$$

P1 finds as its solution the right angled corner (or isoquant) HCJ which is furthest away from the origin yet still part of the feasible region. It is furthest away, not in the sense of Euclidean distance, but so that $C_1/w_1 = C_2/w_2$ is a maximum. Augmenting the maximand changes the isoquant from a right angled corner to the obtuse angled isoquant KCL. Solutions along AB will not be generated, and provided the angles θ_1 and θ_2 are sufficiently small, then all rays that would intersect AB will give B as the optimal solution. The angles θ_1 and θ_2 are a function of both the weights (w_1, w_2) and α. Steuer and Choo provide an upper bound for α in the same way that the upper bound was constructed in Section 4.2.1 concerning inefficient solutions for the extreme solution matrix.

4.7.5 Tradeoff Values - Linear Case

The optimal solution to $P1(\underline{w})$ contains further information which can aid the DM in finding a preferred solution. This information is in the form of pairwise tradeoffs which are found from the Lagrange multipliers of the binding objective constraints.

Theorem

The standard P1 formulation can be written as

$$\begin{aligned} &\text{Max} \quad y \\ \text{s.t.} \quad & f_k(\underline{x}) - w_k y \geq \emptyset \,, \quad k = 1,2,\ldots,q \qquad [25] \\ & \underline{x} \in X \,. \end{aligned}$$

Consider a solution $\underline{f}^o = (f_1^o, f_2^o, \ldots, f_q^o)$ to $Pl(\underline{w})$ with a maximal value y^o, and with all Lagrange multipliers of the objective constraints non-zero, i.e., $\pi_k < \emptyset$, $k = 1,2,\ldots,q$. (In effect, this assumes a properly efficient solution.) The pairwise tradeoff t_{ij} measures the decrease in f_i for a one unit increase in f_j. Then

$$t_{ij} = \delta f_i / \delta f_j = -\pi_j / \pi_i \quad , \; i,j \in \{1,2,\ldots,q\} \qquad [26]$$

This tradeoff is valid in a neighbourhood of the solution $\underline{f}^o$ which, for the linear case, is the current basis.

Proof

Consider an increase in the RHS of objective constraint i by an amount δ_i such that the current basis is unchanged. All objective constraints are binding since $\pi_k < \emptyset$, $k = 1,2,\ldots,q$, and may therefore be written as equality constraints.

$$\begin{aligned} &\text{Max} \quad y \\ &f_k(\underline{x}) - w_k y = \emptyset \quad , \quad k = 1,2,\ldots,q \; , \; k <> i \qquad [27] \\ &f_i(\underline{x}) - w_i y = \delta_i \\ &\underline{x} \in X \end{aligned}$$

The resulting efficient solution $\underline{f}^r$ with value y^r satisfies [27] with

$$\begin{aligned} &f_k^r - w_k y^r = \emptyset \quad , \quad k = 1,2,\ldots,q \; , \; k <> i \qquad [28] \\ &f_i^r - w_i y^r = \delta_i \end{aligned}$$

The value of y^r can be calculated from the Lagrange multipliers (π_k)

$$y^r = y^o + \pi_i \delta_i \qquad [29]$$

with $y^r < y^o$ since $\pi_i < 0$. Therefore the constraints of [28] can be rewritten as

$$f_k^r - f_k^o = w_k \pi_i \delta_i \quad , \quad k = 1,2,\ldots,q \; , \; k <> i \qquad [30]$$
$$f_i^r - f_i^o = \delta_i (w_i \pi_i + 1)$$

since $f_k^o = w_k y^o$ for $k = 1,2,\ldots,q$.

Consider now a decrease in the RHS of objective constraint j (j<>i) of δ_j with the resulting efficient solution being $\underline{f}^t$ with value y^t. The equivalent equations to [30] are

$$f_k^t - f_k^o = -w_k \pi_j \delta_j \quad , \quad k = 1,2,\ldots,q \; , \; k <> j \qquad [31]$$
$$f_j^t - f_j^o = -\delta_j (w_j \pi_j + 1)$$

A pairwise tradeoff t_{ij} requires that the value of all objectives except f_i and f_j be unchanged. Therefore δ_j must be chosen such that for any objective f_p , $p \in \{1,2,..,q\}$ and $p <> i,j$, the decrease in f_p due to δ_i is equal to the increase in f_p due to δ_j,
i.e.,

$$f_p^r - f_p^o + f_p^t - f_p^o = 0 \qquad [32]$$
$$\rightarrow \quad w_p \pi_i \delta_i - w_p \pi_j \delta_j = 0$$
$$\rightarrow \quad \delta_j = \delta_i (\pi_i / \pi_j) \; . \qquad [33]$$

[33] is independent of p and is therefore valid for all $p \in \{1,2,\ldots,q\}$, $p <> i,j$. The total change in f_i is the sum of the increase and the decrease which is

$$\delta f_i = (f_i^r - f_i^o) + (f_i^t - f_i^o) = \delta_i(w_i\pi_i + 1) + -w_i\pi_j\delta_j$$
$$= \delta_i w_i \pi_i + \delta_i - w_i\pi_j\delta_i(\pi_i/\pi_j)$$
$$= \delta_i \ . \qquad [34]$$

And for f_j,

$$\delta f_j = (f_j^r - f_j^o) + (f_j^t - f_j^o) = -\delta_j(w_j\pi_j + 1) + w_j\pi_i\delta_i$$
$$= -w_j\pi_j\delta_i(\pi_i/\pi_j) - \delta_i(\pi_i/\pi_j) + w_j\pi_i\delta_i$$
$$= -\delta_i(\pi_i/\pi_j) \ . \qquad [35]$$

Therefore $\delta f_i/\delta f_j = -\pi_j/\pi_i$, which concludes the proof.

4.7.6 The Range for Tradeoffs

The analysis of this section applies to the linear case since in the non-linear case the tradeoffs are only valid in a neighbourhood of the optimal solution. From the basis inverse at the optimal solution to P1, it is possible to calculate the range for which a tradeoff is valid. Figure 4.5 on the following page is a possible representation of the pairwise tradeoff for two objectives f_i and f_j, with the values of all other objectives unchanged. The figure can be likened to a contour map for only one contour.

At solution $\underline{f}^o$ consider a change in f_j of δ_j. f_i will change by the amount of the tradeoff t_{ij}, $\delta f_i = \delta_j(-\pi_j/\pi_i)$. For $\delta_j > 0$ the tradeoff is valid to extreme point B (and to extreme point C for $\delta_j < 0$). In linear programming the change in the RHS for which a non-zero Lagrange multiplier is valid can be calculated from the columns of the basis inverse (e.g., Daellenbach and George (1978), pp.132-137). From Section 4.7.5 it can be seen that a pairwise tradeoff is equivalent to changing two RHS values simultaneously, i.e., changing the RHS of objective constraint j by δ_j and the RHS of objective constraint i by $-\delta_j(\pi_j/\pi_i)$.

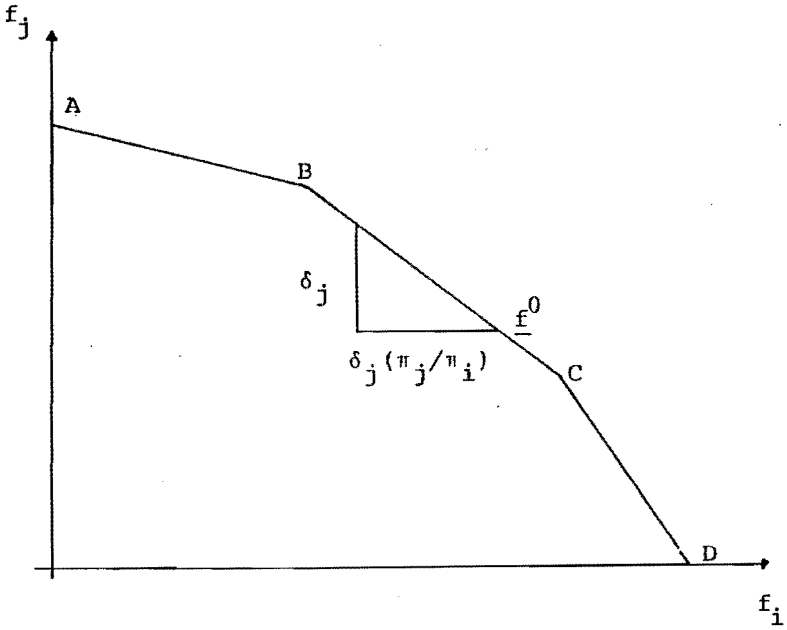


Figure 4.5

Let B^{-1} be the basis inverse at the optimal solution, adjusted so that it corresponds to the initial basis of slack and artificial variables with elements σ_{pi}. Let α_p be the optimal value of the p^{th} basic variable. Then

$$\text{Maximum increase in } f_j = \delta_j^+ = \operatorname*{Min}_p \{ [-\alpha_p / (\sigma_{pj} - (\pi_j/\pi_i)\sigma_{pi}) \quad \text{for } \sigma_{pj} - (\pi_j/\pi_i)\sigma_{pi} < 0 \} \tag{36}$$

$$\text{Maximum decrease in } f_j = \delta_j^- = \operatorname*{Min}_p \{ [+\alpha_p / (\sigma_{pj} - (\pi_j/\pi_i)\sigma_{pi}) \quad \text{for } \sigma_{pj} - (\pi_j/\pi_i)\sigma_{pi} > 0 \}$$

Given that the efficient set is convex, the value of the tradeoff t_{ij} beyond the current basis (line BC in Figure 4.5) will decrease as f_j decreases and vice versa.

Linear combinations of pairwise tradeoffs are also possible <u>within</u> the current basis. In general, an increase in f_j of δ_j is equivalent to a decrease of

$$\begin{array}{ll} \beta_1\delta_j^+(\pi_j/\pi_1) & \text{for } f_1 \\ \beta_2\delta_j^+(\pi_j/\pi_2) & \text{for } f_2 \\ \cdot & \\ \cdot & \\ \beta_q\delta_j^+(\pi_j/\pi_q) & \text{for } f_q \end{array} \qquad [37]$$

where $i <> j$ and $\sum_{\substack{k=1\\k\neq j}}^{q} \beta_k = 1$.

4.7.7 Tradeoffs in the Non-Linear Case

Intuitively, it is reasonable to expect the result concerning pairwise tradeoffs (Section 4.7.5) to hold for the non-linear case. The proof for the case of two objectives is given below.

Given that only q-1 weights are required to define the optimal solution, the P1 formulation be written as follows, dividing through by w_1 and defining $w = w_2/w_1$.

$$\begin{array}{l} \text{Max } \; y \\ f_1(\underline{x}) - y \geq \emptyset \\ f_2(\underline{x}) - wy \geq \emptyset \\ g_j(\underline{x}) \leq \emptyset \quad , \; j = 1,2,\ldots,m \end{array} \qquad [38]$$

At a properly efficient solution $\underline{x}^o$, where all the Lagrange multipliers of the objective constraints are non-zero, the pairwise tradeoff t_{12} is given by

$$t_{12} = \delta f_1/\delta f_2 = -\pi_2/\pi_1 \qquad [39]$$

The Lagrangian of [38] can be written as

$$L = y + \pi_1(f_1 - y) + \pi_2(f_2 - wy) + \sum_{j=1}^{m} u_j g_j$$

At $\underline{x}^o$, $L = y$

$$= f_1 = f_2/w \qquad [40]$$

Therefore

$$\delta L/\delta w = \delta f_1/\delta w \qquad [41]$$

$$= \delta(f_2/w)/\delta w = (\delta f_2/\delta w)/w - f_2/w^2 \qquad [42]$$

Equating [41] and [42]

$$w(\delta f_1/\delta w) - \delta f_2/\delta w = -f_2/w = -y \qquad [43]$$

Also,

$$\delta L/\delta y = \emptyset = 1 - \pi_1 - w\pi_2$$

$$\longrightarrow w = (1-\pi_1) / \pi_2 \qquad [44]$$

Substituting [44] in [43] gives

$$\delta f_1/\delta w - \pi_1(\delta f_1/\delta w) - \pi_2(\delta f_2/\delta w) = -\pi_2 y$$

$$= \delta f_1/\delta w$$

since $\delta L/\delta w = -\pi_2 y$.

$$\longrightarrow \pi_1(\delta f_1/\delta w) + \pi_2(\delta f_2/\delta w) = \emptyset$$

$$\longrightarrow (\delta f_1/\delta w) / (\delta f_2/\delta w) = \delta f_1/\delta f_2 = -\pi_2/\pi_1 \qquad [45]$$

This completes the proof for the two objective non-linear case.

While the results of the three previous sections have been based on the P1 formulation, these results also hold for the P2 (minmax) formulation.

The following relationships also hold for the pairwise tradeoffs. For $i,j,k \in \{1,2,...,q\}$ and $i <> j <> k$,

$$t_{ij} = -t_{ik} \cdot t_{kj} \quad , \quad t_{ij} = 1/t_{ji} \qquad [46]$$

The proof follows from the definition of $t_{ij} = -\pi_j/\pi_i$.

4.7.8 A Tradeoff Solution Method (TO)

These results on tradeoffs and their ranges can be incorporated into an interactive solution method. An approach which utilizes these results will be examined in some detail.

In essence, this tradeoff method (TO) aims to allow the DM to make a pairwise tradeoff at each iteration, thereby moving over the efficient surface to a more preferred solution. The method is, in principle, similar to the interactive surrogate worth tradeoff method (ISWT) of Chankong and Haimes (1978) in that the DM effectively specifies a direction of improvement and a step size at each iteration. This similarity in principle is not to be unexpected, given the similarity of the P1 and P2 formulations with the e-constraint formulation as is used in the ISWT method. Sections 4.8 and 4.9 will illustrate these similarities. This TO method also exhibits a number of similarities with Wierzbicki's (1980) reference point approach (see Sections 2.3.2.5 and 6.1.4.3).

Figure 4.6 on the following page details the solution method for two objectives.

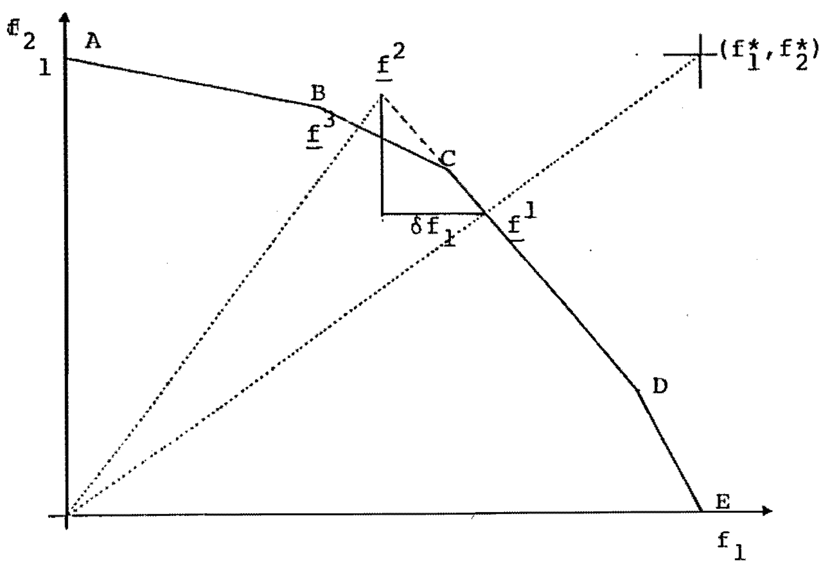


Figure 4.6

An outline of the method is as follows.

1. Choose the ideal solution as the first guess which gives the initial solution $\underline{f}^1 = (f_1^1, f_2^1)$.

2. Present to the DM the pairwise tradeoff $\delta f_2/\delta f_1 = t_{21}$ which is the slope of CD. Using this tradeoff the DM chooses the desired change in f_1 (δf_1) and a new solution is calculated.

$$\underline{f}^2 = (f_1^1+\delta f_1^1 \; , \; f_2^1+t_{21}\delta f_1) = (f_1^2, f_2^2)$$

3. Using the new solution $\underline{f}^2$ as the weights in P1, the resulting actual solution $\underline{f}^3$ is calculated. If the tradeoff goes beyond the current basis (i.e., line CD), the new desired solution $\underline{f}^2$ will be infeasible. Therefore this method will perform better for only small changes in the objectives at each iteration.

4. If the DM is satisfied with the resulting solution then the process terminates. Otherwise, go to Step 2 and continue.

This is the method in outline form where the basic concept is to allow the DM the freedom to move anywhere over the efficient surface through the use of pairwise tradeoffs.

4.7.8.1 Comparison of P1 and P2 formulations for use in the method

While either P1 or P2 can be used as the solution mechanism in the method, they do exhibit different "operating characteristics".

Again a simple analysis will be done in two dimensions. Consider Figure 4.7 below where the objectives have been normalized to [Ø,1] (range norm).

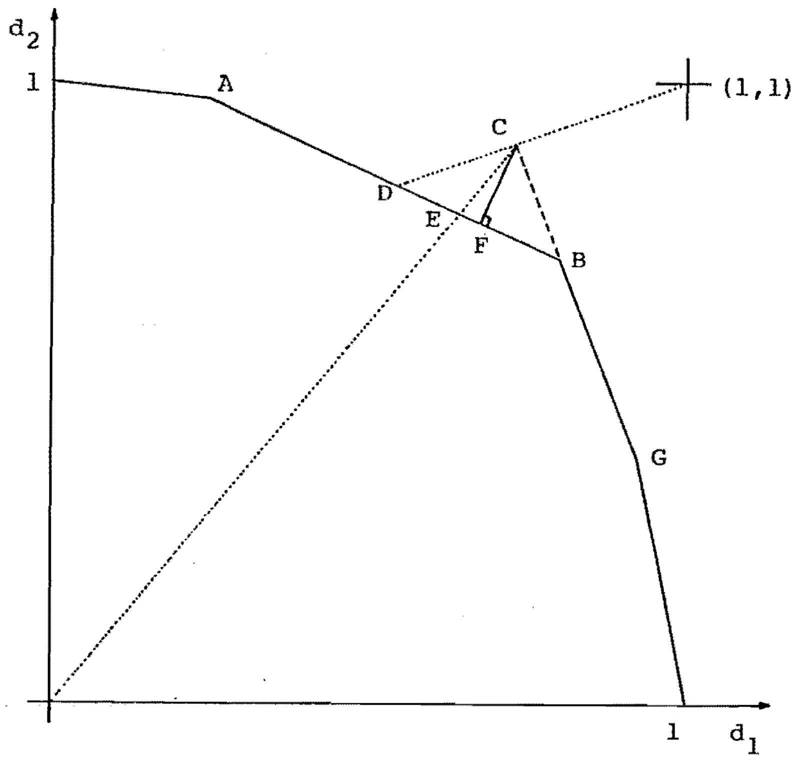


Figure 4.7

The figure represents the following situation:
A tradeoff has occurred along GB to point C. Using the solution at C as weights, P1 gives solution E and P2 solution D on the line AB. Solution F is closest (in terms of Euclidean distance) to the efficient surface from C.

In order to ascertain which formulation is on average closer to the efficient surface, this situation was simulated for random lines AB. The results show that for a sample size of 3000, P1 was closer (in terms of Euclidean distance) to the efficient set than P2 81% of the time. (Appendix 2 contains the details of the simulation). This figure of 81% can be tempered somewhat by considering that although P2 will, at the edges of the efficient set, give solutions a long way from the closest point, most decision making will probably take place nearer the middle where the discrepancy will be smaller.

A further and more important consideration is the inability of P1 to generate correct solutions in certain situations. Consider Figure 4.8 on the following page.

In case (a) there are no difficulties; a ray from the origin defined by weights $\underline{w} = (-1,3)$ gives the following objective constraints

$$f_1 + y \geq 0$$

$$f_2 - 3y \geq 0$$

These have as their optimal solution $\underline{f}^* = (-1,3)$ with $y^* = 1$. However in case (b), given the shape of the efficient set, and using the same set of weights $\underline{w} = (-1,3)$, the optimal solution is $\underline{f}^* = (-2,9)$ with $y^* = 3$. It is almost as if the ray hits the smooth outer surface of the efficient set and slides along it as far as possible.

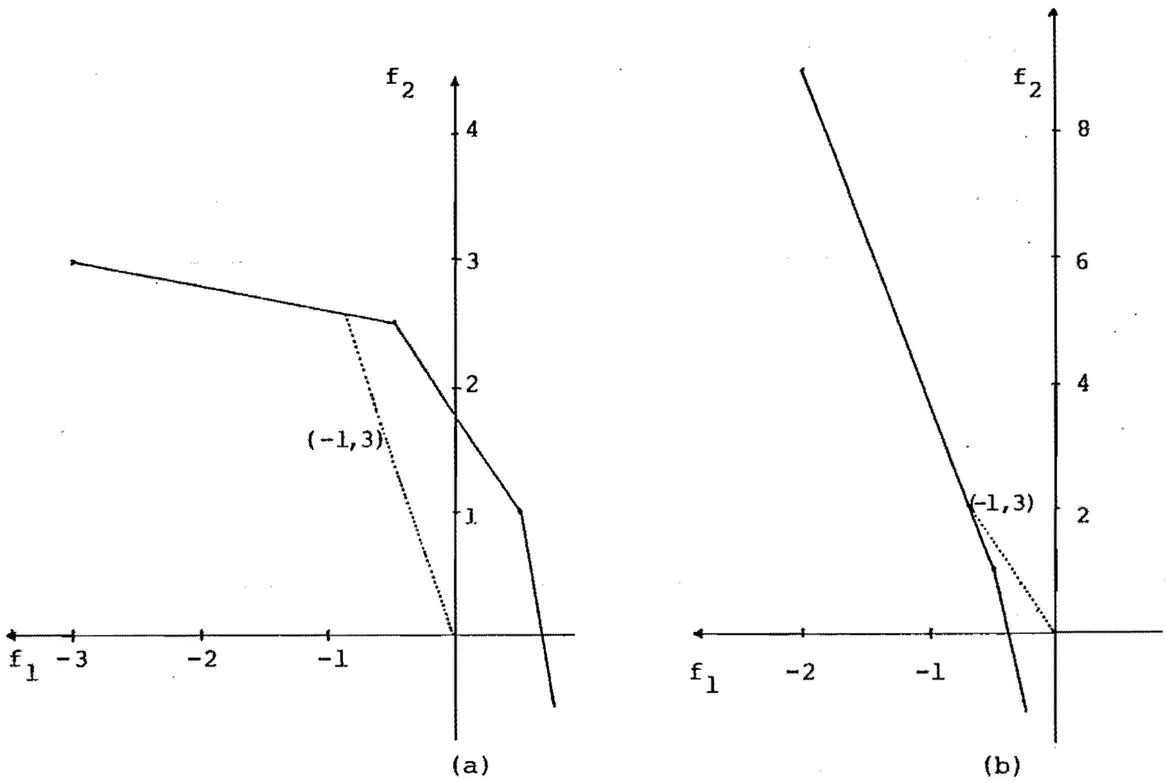


Figure 4.8

Effectively, what is happening when one of the weights is negative, is that the negatively weighted objective may play no part in determining the optimal solution. Therefore with more than two objectives, this situation generally does not arise since there are usually at least two objectives with non-negative weights which will constrain the solution. Also, practical experience with the P1 formulation (with three or more objectives) has not revealed any problems in having one negative weight.

This can be overcome by transforming the entire objective space into the positive quadrant. Then the ray from the origin will always approach the efficient surface from the "inside". The problem becomes

$$\begin{aligned}
&\text{Max } y \\
&f_k(\underline{x}) - w_k y \geq -M_k \quad , \quad k = 1,2,\ldots,q \qquad [47] \\
&\underline{x} \in X \\
&\qquad\qquad\qquad \text{for } M_k \leq \emptyset
\end{aligned}$$

If $M_k \geq \emptyset$, the RHS of the objective constraints remain at Ø.

However, since the true value of M_k is generally not known exactly, the transformation may not be entirely into the positive quadrant, and therefore the situation with negative weights may still persist. Multiplying $\hat{M}_k$, as found from the matrix of extreme solutions, by a factor (1+e) where $\emptyset < e \ll 1$, should overcome this difficulty.

In contrast, when cases (a) and (b) are considered under the minmax formulation P2, it can be seen that this situation with the negative weights does not occur. Since it is only deviations from the ideal that are being measured, the weights will never become negative.

4.8 COMPARISON WITH THE E-CONSTRAINT FORMULATION

It is clear that the properties of the P1 (or P2) formulation are similar to the e-constraint formulation as used in the SWT method. Both are capable of generating every efficient solution and can provide pairwise tradeoff information at every properly efficient solution. For comparison the two formulations are

$$\begin{aligned}
&\qquad \text{Max } y \\
&P1(\underline{w}) \qquad f_k(\underline{x}) - w_k y \geq \emptyset \quad , \quad k = 1,2,\ldots,q \qquad [48] \\
&\qquad \underline{x} \in X
\end{aligned}$$

$$P(\underline{e}) \qquad \begin{array}{l} \text{Max } f_1(\underline{x}) \\ f_k(\underline{x}) \geq e_k \qquad , \quad k = 2,3,\ldots,q \\ \underline{x} \in X \end{array} \qquad [49]$$

The main difference is in the mechanics of solution. Both $\underline{w}$ and $\underline{e}$ can be considered as solution values desired by the DM. P1($\underline{w}$) seeks to preserve the <u>relative</u> values of the desired solution in finding an efficient solution, while P($\underline{e}$) preserves the values of the desired solution <u>absolutely</u> (unless infeasible). P1($\underline{w}$) is like the laser beam mounted at the origin which illuminates a single efficient solution with the direction of the beam determined by $\underline{w}$. In contrast, P($\underline{e}$) reduces the size of the objective space by adding a number of cutting hyperplanes determined by $\underline{e}$.

4.9 A SMALL MOLP EXAMPLE

An example of a MOLP modified from Zionts and Wallenius (1976) will be used to illustrate some of the properties of the P1 formulation discussed in previous sections and also to highlight the similarities with the e-constraint formulation. The modified example is

$$\begin{aligned} \text{'Max'} \quad f_1(\underline{x}) &= 3x_1 + x_2 + 2x_3 + x_4 \\ f_2(\underline{x}) &= x_1 - 2x_2 + 2x_3 + 4x_4 + 35 \\ f_3(\underline{x}) &= -x_1 + 8x_2 + 1.6x_3 + 3x_4 + 25 \end{aligned} \qquad [50]$$

$$\text{s.t.} \quad X = \{x : \begin{aligned} 2x_1 + x_2 + 4x_3 + 3x_4 &\leq 60 \\ 3x_1 + 4x_2 + x_3 + 2x_4 &\leq 60 \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned} \;\}$$

There are 9 extreme points, and 6 of these are efficient.

Table 4.1 below contains the extreme points.

	x_1	x_2	x_3	x_4	f_1	f_2	f_3
A	18	Ø	6	Ø	66	65	16.6
B	12	Ø	Ø	12	48	95	49
C	Ø	12	12	Ø	36	35	14Ø.2
D	Ø	6	Ø	18	24	95	127
E	Ø	15	Ø	Ø	15	5	145
F	Ø	Ø	Ø	2Ø	2Ø	115	85
G	2Ø	Ø	Ø	Ø	6Ø	55	5
H	Ø	Ø	15	Ø	3Ø	65	49
I	Ø	Ø	Ø	Ø	Ø	35	25

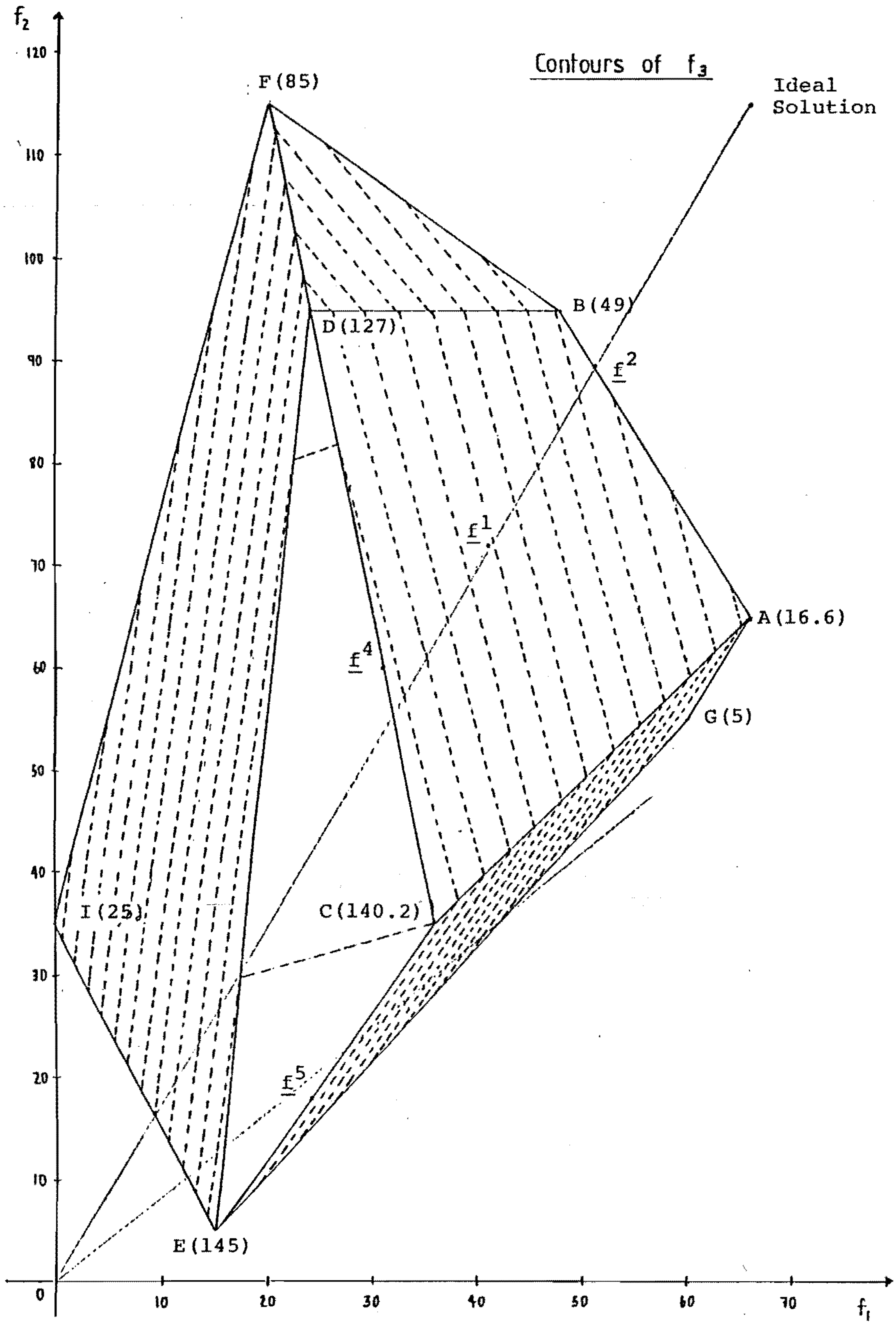
<u>Table 4.1</u>

The following diagrams (Figures 4.9 and 4.1Ø) give the shape of the efficient set both in terms of a two dimensional contour map and as an isometric three dimensional projection. Faces BDF and ABCD are efficient, as also is edge EC. The inefficient faces can be identified by positive sloping contour lines, i.e., it is possible to move along a contour and increase two objectives simultaneously. More precisely, solutions on the edge of the efficient faces are not properly efficient, while those "inside" the faces BDF and ABCD are properly efficient.

The P1 and e-constraint formulations of [5Ø] follow directly from [48] and [49] where f_3 is the maximand in $P(\underline{e})$.

<u>4.9.1 Properly Efficient (PE) Solutions</u>

Consider the P1 formulation with $\underline{w}$ as the ideal solution, $\underline{w}_1 = \underline{f}^* = (66,115,145)$. This is a reasonable choice for the DM's desired solution, for if it is achieved then there exists a single solution which simultaneously maximizes all objectives and the MOLP problem is solved. However in this



f_2
120
110
100
90
80
70
60
50
40
30
20
10
0
10
20
30
40
50
60
70
f_1
Contours of f_3
Ideal Solution
F(85)
B(49)
D(127)
A(16.6)
G(5)
I(25)
C(140.2)
E(145)
$\underline{f}^2$
$\underline{f}^1$
$\underline{f}^4$
$\underline{f}^5$

Figure 4.9

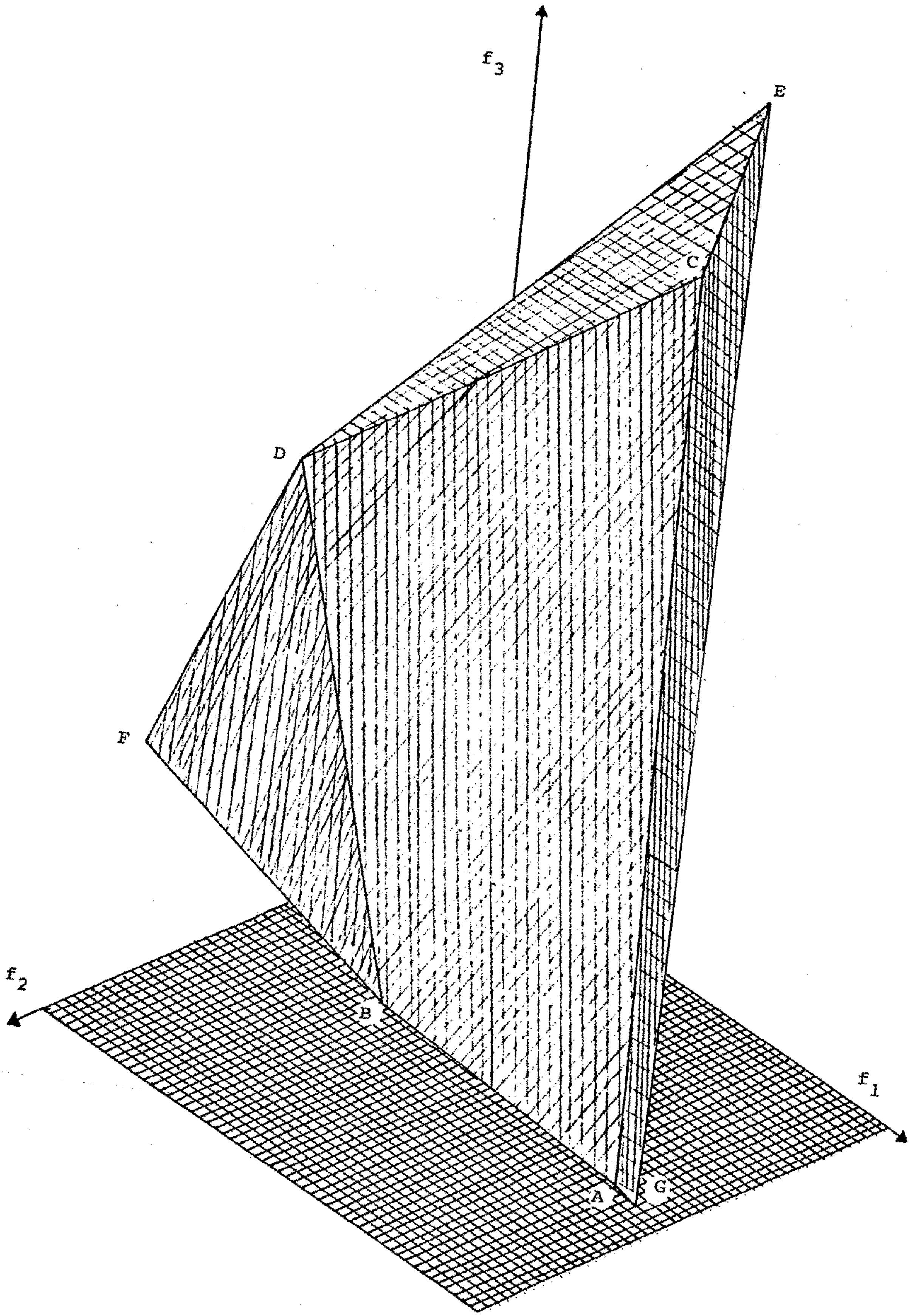


Figure 4.10

case, the desired solution is infeasible and therefore P1 seeks a solution which has the same relative proportions of each objective as the desired solution. The resulting solution is $\underline{f}^1 = (41.31, 71.98, 90.76)$ with $y^1 = 0.626$, i.e., a 62.6% achievement of each objective. The relative proportions of each objective have been preserved, since the initial guess was for 100% achievement of each objective. This is a sufficient condition for the resulting solution to be PE.

Also at this solution, the values of the Lagrange multipliers are

$$\pi_1 = -7.072E-3 \ , \ \pi_2 = -1.893E-3 \ , \ \pi_3 = -2.176E-3 \ .$$

The tradeoff t_{12}, which is the amount of f_1 required to be given up to achieve an additional unit of f_2, is

$$t_{12} = \delta f_1/\delta f_2 = -\pi_2/\pi_1 = -0.268$$

This is the inverse of the slope of the f_3 contour line at $\underline{f}^1$.

The range for this tradeoff can be calculated from the basis inverse at $\underline{f}^1$. After adjusting to standard form (i.e., maximize with "$\leq$" constraints) the two columns of the basis inverse which correspond to objective constraints 1 and 2 are given in Table 4.2 below.

	f_2	f_1	RHS
x_1	0.0158	0.1853	6.354
x_2	-0.0861	-0.0113	5.125
x_3	-0.1565	0.1627	4.603
x_4	0.2268	-0.3366	7.918
y	-0.0019	-0.0071	0.626

Table 4.2

Then,

$$\text{Max increase} = \text{Min}\left\{\frac{-4.603}{-0.1565-(0.268)(0.1627)}, \frac{-6.354}{0.0158-(0.268)(0.1853)}, \frac{-5.125}{-0.0861+(0.268)(0.0113)}\right\}$$

$$\text{in } f_2 \quad = \quad 23.01$$

$$\text{Max decrease} = \text{Min}\left[\frac{7.918}{.2268+(0.268)(0.3366)}, \frac{0.626}{-0.0019+(0.286)(0.0071)}\right\}$$

$$\text{in } f_2 \quad = \quad 24.98$$

Thus f_2 can be increased to $71.98 + 23.01 = 94.99$ (edge DB) or decreased to $71.98 - 24.98 = 47$ (edge CA).

Similar information can be derived from the e-constraint formulation, $P(\underline{e})$. Letting $\underline{e} = (41.31, 71.98)$ gives the same solution, with the Lagrange multipliers for the constrained objectives being $\lambda_1 = -3.25$ and $\lambda_2 = -0.87$. Therefore $t_{31} = -3.25$ and $t_{32} = -0.87$. Using the results of Haimes et al.(1975), the tradeoff t_{12} can be obtained.

$$t_{12} = -t_{13}.t_{32} = -t_{32}/t_{31} = -0.268.$$

In this case the range information for t_{12} is not readily available, since the standard range information gives the maximum increase and decrease in f_2 for the tradeoff t_{32}, not for t_{12}.

A further distinction between $P1(\underline{w})$ and $P(\underline{e})$ is that in the latter, some choices of $\underline{e}$ will result in no feasible solution, whereas P1 will be feasible for any $\underline{w}$.

4.9.2 Not Properly Efficient (NPE) Solutions

It is possible to have a solution which is efficient but not PE; it is on an edge rather than inside an efficient face. Consider a desired solution of $\underline{w}^2 = (66, 115, 20)$ which indicates that f_3 is much less important than f_1 or f_2.

The solution to $P1(\underline{w}^2)$ is $\underline{f}^2 = (51.33, 89.44, 43.0)$ with $y^2 = 0.778$, which is on the edge AB. Along the ray (66,115) in f_1, f_2 space it is not possible for f_3 to be less than 43.0, and still remain feasible. At $\underline{f}^2$, $f_3^2/w_3^2 = 43.0/20 = 2.15$, which is greater than the maximal value of y. This means that there is no solution with the same proportions of each objective as in the desired solution $\underline{w}^2$. These proportions will only be maintained if the ray defined by $\underline{w}$ intersects the efficient set, and in this case $\underline{w}^2$ does not intersect the efficient set.

The Lagrange multipliers at $\underline{f}^2$ are $\pi_1 = -7.407E-3$, $\pi_2 = -4.444E-3$, $\pi_3 = 0$. These values give no information about the pairwise tradeoffs, only the total tradeoff is defined.

$$T_{12} = -\pi_2/\pi_1 = -0.6$$

This is the inverse of the slope of the edge AB. It is a total tradeoff because as one moves along this edge, f_3 also changes.

However, as can be seen from Figure 4.9, there do exist pairwise tradeoffs at this point, albeit in one direction only. This pairwise information can be obtained by using a different weighting vector

$$\underline{w}^3 = (f_1^2, f_2^2, f_3^2+\delta_3) \quad , \quad 0 < \delta_3 \ll 1$$
$$= (51.33, 89.44, 43.05) \quad , \text{ with } \delta_3 = 0.05 \text{ .}$$

If δ_3 was identically zero this would in theory give exactly solution $\underline{f}^2$, again with $\pi_3 = 0$. In consideration of this and the reality of rounding error in LP codes, the use of a small positive value for δ_3 will bring the solution just onto the efficient face ABCD, i.e., to a PE solution where all pairwise tradeoffs and their valid ranges can be assessed. The solution to $P1(\underline{w}^3)$ is

$\underline{f}^3 = (51.32, 89.43, 43.04)$ with

$\pi_1 = -11.297E-3$, $\pi_2 = -3.024E-3$, $\pi_3 = -3.476E-3$. Thus $t_{12} = -\pi_2/\pi_1 = -0.268$ as calculated for the first solution; and it is only valid for decreases in f_2.

Similarly, the e-constraint formulation also can generate NPE solutions. For example, $\underline{e}^4 = (10,60)$ gives the solution $\underline{f}^4 = (31,60,134.7)$ with $\lambda_1 = 0$ and $\lambda_2 = -0.22$. The only tradeoff information available at this solution is that the total tradeoff $T_{32} = -0.22$, which is the slope of the line CD in f_2, f_3 space. Again, the $\underline{e}$ vector can be perturbed to $(31+\delta_1, 60)$ in order to obtain pairwise tradeoff information at this point.

4.9.2.1 Solutions on a Lesser Dimensional Face

All points on edge EC, which is a line in three dimensional space, are NPE. There are no pairwise tradeoffs at any point on the edge; all three objectives vary simultaneously. And unlike the previous case, perturbing the weight vector $\underline{w}$ will not provide pairwise tradeoff information.

To illustrate this, consider $\underline{w}^5 = (30,25,200)$. The solution to $P1(\underline{w}^5)$ is $\underline{f}^5 = (24.01, 17.87, 142.94)$ with objective constraint 1 slack . $\underline{f}^5$ is on the ray $(w_2, w_3) = (25,200)$, not on the ray described in f_1, f_2 space by $(w_1, w_2) = (30,25)$. Perturbing the weight vector to $\underline{w}^6 = (24.01+0.01, 17.87, 142.94)$ gives an almost identical solution $\underline{f}^6$ with objective constraint 2 slack. It is possible to ascertain total tradeoff information from the Lagrange multipliers of these two solutions.

$$\pi_1^5 = 0 \qquad \pi_1^6 = -1.593E-3$$
$$\pi_2^5 = -0.784E-3 \qquad \pi_2^6 = 0$$
$$\pi_3^5 = -4.902E-3 \qquad \pi_3^6 = -6.737E-3$$

$$T_{32}^5 = -\pi_2^5/\pi_3^5 = -0.16 \qquad T_{13}^6 = -\pi_3^6/\pi_1^6 = -4.375$$

which combines to give

$$T_{12} = -T_{13}^6 . T_{32}^5 = -0.7$$

which is the negative of the inverse of the slope of EC.

Edge EC is a hyperplane of lesser dimension than the original objective space. The occurence of such a hyperplane can be identified by oscillations among the objective constraints between slack and binding as the weighting vector is perturbed. These oscillations distinguish this form of NPE solution from that of a solution which is at the edge of a PE surface, where pairwise tradeoff information can be obtained by perturbing.

The e-constraint formulation behaves in a similar fashion as the $\underline{e}$ vector is perturbed. However Haimes and Chankong (1979) do address this situation when one or more of the Lagrange multipliers equals zero. They consider a solution $\underline{x}^* \in E$ to $P(\underline{e}^*)$ with

$$\lambda_j < 0 \quad , \quad j = 2,3,\ldots,p$$
$$\lambda_k = 0 \quad , \quad k = p+1,\ldots,q$$

A change in f_j of δ_j will result in a change to f_1 (assuming f_1 is the maximand) of $\lambda_j\delta_j$ and a change in every f_k of

$$\delta f_k = \text{grad}\ (f_k(\underline{x}^*)).(dx(\underline{e}^*)/de_k).\delta_j\ ,\ k = p+1,\ldots,q. \quad [51]$$

$x(\underline{e})$ is a function, valid in a neighbourhood of $\underline{x}^*$, which returns the x values for a given vector $\underline{e}$. It is very difficult to analytically evaluate $x(\underline{e})$ even for a small MOLP problem; consequently the result is not of much practical use.

Perhaps it is to avoid such problems that Chankong and Haimes (1983a, p.10) state that "naturally one should only seek, as candidates for the best compromise solution, properly efficient solutions". This is a restrictive assumption. Even for this small example it is not difficult to provide a concave utility function where the solution of maximum utility lies on the edge EC . For example,

$$U(f_1,f_2,f_3) = -[(100-3f_1)^6 + (100-5f_2)^6 + (100-0.68f_3)^6]^{(1/6)}$$

which is maximized at $\underline{f}^7 = (24.38,18.39,142.9)$ with U= -26.878 . Also, for a MOLP which has a large number of these lesser dimensional faces, many potential efficient solutions will be ignored.

In illustrating the similarities between the P1 and the e-constraint formulation, it can be seen that both provide the same information and suffer from the same drawbacks. One observation can be made in favour of P1 in the light of the SWT method. Since P1 never gives infeasible solutions, it may be more useful as a solution generating mechanism than $P(\underline{e})$ in the SWT method.

4.10 CONCLUSION

This chapter has investigated in some detail the properties of the P1 formulation and outlined two possible MOLP solution methods which have resulted from the analysis. The similarity between P1($\underline{w}$) and P($\underline{e}$) has also been illustrated. Chapter 6 develops further the tradeoff solution method already discussed and introduces another solution method which is also based around the P1 formulation.

CHAPTER 5 A COMPARATIVE EVALUATION OF FOUR MODM SOLUTION METHODS

5.Ø INTRODUCTION

As seen from the literature review of Chapter 2, a number of different solution methods for MODM's have been developed. However, with the notable exception of goal programming, there are few solution methods which can cite a large number of real world applications. Consequently, it is difficult to arrive at an assessment of the usefulness of these methods in a practical situation. And it is even more difficult to ascertain user preferences among a number of different solution methods. This is because the users of a solution method will generally have experience only with that particular method. It is possible that, after a number of years, the extent to which individual solution methods are being used in practical decision making situations will provide a measure of performance. But even this could be largely determined by the extent to which individual methods are "marketed" to the users.

The real difficulty stems from the fact that there is no objective basis for comparing one MODM solution method against another. In non-linear programming, for example, a number of different solution methods can easily be compared using such objective measures as seconds of process time and closeness to the true optimal solution. But for MODM solution methods, since the final solution is determined from subjective preference information, objective standards of comparison are generally not available.

One of the best approaches for comparing MODM solution methods could be in the context of a business game, [e.g., de Samblanckx et al (1982)]. They describe the ORSIAM business game where groups of six students form competitive firms and then run each firm over a number of periods. In their experiment each student group initially made their period one decisions unaided and then later made these decisions again using the method of Zionts and Wallenius, within the context of a suitable decision model. They found that use of the decision model and the Zionts Wallenius solution method resulted in better scores than for the earlier unaided decisions. A similar such experiment could be set up with some student groups making unaided decisions and others having a particular MODM solution method available to aid their decision making. The objective measure, by which different solution methods could be compared, would be something like the total profit over all periods. Such an experiment would need to be repeated a number of times for the results to be statistically valid, and over time it should become clear if some MODM solution methods are consistently resulting in better decisions than others.

Wallenius (1975) conducted an experiment in which three different MODM solution methods were compared. The solution methods were; the method of Geoffrion, Dyer and Feinberg, the STEM method (both described in Chapter 2) and the very simple naive method mentioned in Section 4.7.3 . This chapter details an experiment very similar to that of Wallenius', where four different MODM solution methods are compared. A case study of a "pseudo real" problem was given to each subject, who after adequate familarization with the problem, gained actual experience in finding a solution to it using each of the four solution methods. The experiment was motivated by a desire to see if some methods were actually preferred to others and also if the results of the experiment would be consistent with the behavioural analyses of Chapter 3.

5.1 THE CASE STUDY

The actual case study which was given to each subject is contained in Appendix 3 (A3.1). It details a small, fictitious New Zealand manufacturing company which produces electrical components for use in lamps. A linear programming model describing feasible production schedules was derived, having three objectives. The three objectives to be minimized are; operating costs, the average number of stockouts and the average percentage of the labour force temporarily laid off. This case study is the same as that used by Wallenius in his experiment, except that both the description of the company and the MOLP have been modified for the New Zealand situation. The details of the MOLP are also given in Appendix 3 (A3.2).

Four different planning situations were derived from the MOLP. These situations or scenarios are called normal, pessimistic, optimistic and conservative, and each is a MOLP of around 20 rows and 25 columns.

5.2 THE SOLUTION METHODS.

Each of the four solution methods used in the experiment will be described below. However since the methods have been described in some detail in previous chapters, this description will only be concerned with those details not already covered. A demonstration of each method in operation using the normal planning situation is given in Appendix 3 (A3.4). This will clearly show the details of each method. While this demonstration is somewhat lengthy, it has been included so that the cosmetics of presentation for each solution method will be evident.

Each method has been subjected to substantial testing before being used in the experiment, and the author is confident that each method performs in a manner consistent with the intent as described in the original papers.

5.2.1 The Method of Zionts and Wallenius (1976,1982)

The implementation of the ZW method follows the explanation of Section 2.3.2.3 and Zionts and Wallenius (1976). However, with regard to the presentation of the tradeoffs to the DM, the approach of Zionts and Wallenius (1982) has been adopted. Instead of tradeoffs being presented in terms of the tradeoff vector $\underline{w}$, they are presented to the DM in terms of two adjacent efficient solutions. These two solutions are the current efficient extreme point solution and the adjacent efficient extreme point solution which would result from pivoting in the appropriate non-basic vector.

The only difficulty with this approach is that sometimes the two adjacent solutions are so close as to render comparison very difficult. However, this was deemed to be preferable to the alternative situation where sometimes the DM was presented with tradeoffs and sometimes with adjacent solutions. In the experiment the DM's were encouraged not to use the response of "indifferent" unless absolutely necessary.

The ZW method was the most difficult of the four methods to program, with the MOLP models having to be rescaled. The scaling problem occurred in the constraints that determined a feasible weighting vector consistent with previous choices. Some experimentation was necessary to find a suitable value for e , the RHS of the constraints [31] in Chapter 2. Also, this method requires information from "inside" the LP code, e.g., the basis inverse is needed to update a non basic vector so that it can be pivoted in to find an adjacent solution. This is not difficult with simple LP codes, but may become more so with the larger commercial LP codes.

5.2.2 The Naive Method

This method is exactly as explained in Section 4.7.3 . The DM specifies a desired solution and is given an achieved solution which is efficient, whose individual objective values should be in about the same proportions as in the desired solution. Based on the information contained in the achieved solution, the DM chooses another desired solution and the method terminates when the DM is satisfied with a given achieved solution. A record or log of all achieved solutions is also available.

5.2.3 The SWT Method

As already mentioned in Section 2.3.1.2, there are a number of possible approaches for utilizing the information contained in the surrogate worth values in order to find the most preferred solution. Clearly, if there exists a single solution for which all surrogate worth values are zero, then this gives the most preferred values for the q-1 constrained objectives. Because the method of using multiple regression to find an indifference solution was not especially successful, being too much of a "black box" approach, the more intuitive (and interactive) approach of Goicoechea et al. (1982, Section 4.4) was adopted.

In this approach the analysis is performed in a truly pairwise fashion with only two objectives ever considered at any stage. Initially a reference objective is chosen, which for the case study was operating costs. All objectives except the reference objective (f_r , $r \in \{1,2...,q\}$) and one other objective (f_j , $j \in \{ 1,2,...,q\}$, $j \neq r$) are then fixed at values determined by the analyst. In the experiment a midpoint value was used which is approximately $(U_k+M_k)/2$ for any objective f_k , $k \in \{1,2,...,q\}$, $k \neq r,j$. Pairwise tradeoffs on the remaining two objectives, f_r and f_j are assessed by the DM who provides worth values for each tradeoff. After assessing seven tradeoffs, a plot of the worth values against f_j is presented to the DM, who is then given the opportunity to change any assessments, if so desired. From this worth information a simple linear regression is performed to find a point of indifference and therefore the most preferred level of f_j . f_j is then constrained at this level.

This procedure is repeated q-1 times as the most preferred levels of each of the q-1 objectives is found at the point of indifference. Finally, the reference objective is optimized with all the other q-l objectives constrained at their most preferred levels, and the method terminates.

While this approach is quite intuitive, it can become cumbersome and it is possible that more than q-l iterations may be necessary. For example, in the case study labour was initially fixed and cost/stockout tradeoffs were presented to the DM. Stockouts were then fixed with cost/labour tradeoffs presented to the DM. At this point it may be appropriate to rework the cost/stockout tradeoffs <u>again</u> before constraining both stockout and labour and optimizing the cost objective.

Another potential difficulty was the generation of a number of properly efficient solutions to present to the DM. Inappropriate choices of RHS values for the constrained objectives can give infeasible solutions. Fortunately, with the MOLP's of the case study this did not happen frequently.

5.2.4 The Method of Steuer and Choo (1983), STE

This method follows exactly the outline of Section 2.2.2.4 and the article by Steuer and Choo, with the exception that the maxmin formulation was used instead of the minmax formulation. Initially six efficient solutions are presented to the DM who is asked to choose one. These six solutions have been generated randomly from within the gradient cone defined by the objectives; and by using a filtering process, they should be representative of the entire set of efficient solutions. Specifically, 100 randomly generated sets of weights are filtered to give twelve distinct sets; using each set twelve MOLP's are solved, and these twelve solutions are then filtered to give the final six solutions. The gradient cone is then reduced centering around the solution chosen by the DM and another six representative solutions are presented to him or her. The process terminates when the DM is satisfied with the current chosen solution. At each stage the cone is proportionately reduced in size, therefore after a large number of iterations all six solutions presented to the DM will be almost identical.

The STE method uses the P1 formulation to generate the efficient solutions and a filtering routine from the ADBASE package [Steuer (1983)] to find the subset of most distinct solutions.

5.2.5 Details of the Computer Code

The LP code which was used in every solution method uses a full tableau (excluding slack and surplus variables) which is updated at every iteration. No attempt has been made to write an efficient code for each solution method; however all four solution methods are at the same standard of efficiency, e.g., every solution is performed starting with the initial tableau, rather than updating from the basis inverse. Consequently, the CPU times for each method are comparable.

All codes are written in standard FORTRAN IV and run interactively on a Burroughs B6900 using the CANDE system.

5.2.6 Choice of Methods

The reasons for choosing these four methods are as follows. The Naive method has been included as a base for comparison, because it is the simplest of all the four methods. The other three methods are among the most well known within the MODM literature (for continuous decision variables) and all can cite examples of practical application.

Also, these methods differ considerably with regard to underlying principles, termination criteria, and the type of response required from the DM. SWT makes use of pairwise tradeoffs and requires cardinal information from the DM, while ZW uses total tradeoffs and only requires a response of "yes", "no" or "indifferent". ZW requires only pairwise assessments whereas STE requires the DM to choose among six solutions at each iteration. As regards convergence to a most preferred solution, both ZW and SWT will converge under an ideal DM, and STE should converge. The Naive method is not designed to converge in any predetermined fashion. Both ZW and SWT terminate "internally", while STE and the Naive method terminate when the DM is satisfied with the current solution, i.e., "externally". Finally, STE is a global approach, attempting to start from outside and work in to a preferred solution. In contrast, ZW and the Naive method represent a local approach where the DM never really gets to see the "whole picture", and SWT is somewhere in between.

Given all the differences in these methods of solution, it is not unreasonable to expect different DM's to prefer one method over another. And since it is expected that different DM's will exhibit different decision making behaviour, the null hypothesis for this experiment is that over a number of DM's there will be no significant difference between the methods in terms of DM preference.

5.3 EXPERIMENTAL DESIGN

5.3.1 Criteria for Measuring Performance

As stated above, the purpose of this experiment is to test whether the four solution methods are significantly different from each other. A number of different criteria have been chosen on which to measure the performance of the methods; again the approach is similar to that of Wallenius' experiment. These criteria are

1. DM's confidence in the final solution. This measures how confident the DM is that the solution method which was used actually enabled him or her to find a satisfactory solution. It was stressed to each DM at the beginning of the experiment that this measure should be independent of the actual planning situation (i.e., normal, pessimistic, etc.). Otherwise much higher ratings would possibly be given under the optimistic planning situation than under the pessimistic situation.

2. Ease of use of the method

3. Ease of understanding the logic of the method. This understanding is expected to come from two sources. Firstly from an initial description of the method before the method was used, and secondly from participating in the actual solution process.

4. CPU time (seconds)

5. Elapsed time (minutes). This measures the total time from

the start of the method to termination.

6. Relative preference for using each method.

This final criterion is certainly the most important one. It concerns the situation where the DM actually has to use one of these methods in a practical decision making situation and, as such, it should implicitly include all the other five criteria.

The means by which this information was extracted from each DM differs from that used by Wallenius (1975). Each DM was required to assess a particular solution method on the first three criteria immediately after using the method. This was done by placing a mark on a line where the endpoints are defined. The actual form of this questionnaire is given in Appendix 3 (A3.5). The alternative (as adopted by Wallenius) is to have the DM rank all four methods on each criteria at the end of the session.

There are two distinct differences between these approaches. The first concerns the point in time when the DM provides his or her assessment, and the second deals with the type of information provided by the DM. Both will be discussed with the timing aspect considered first.

In the approach where a solution method is assessed immediately after use, there is the possibility of a scaling problem which may hinder the aggregation of all scores for a given criterion. For example, if the assessment of the first solution method by a subject is reasonably high and all subsequent methods score even higher, then that subject effectively uses only part of the scale. His or her judgement becomes anchored by the first assessment. However, this anchoring effect can also occur in the other approach, i.e., where all solution methods are scored at the end of the experiment. In this case, the more recently used solution methods will be the most salient or prominent in memory, and may therefore bias the assessment against the first or second solution method used[1].

The two approaches also differ with respect to the information provided. The former approach, in obtaining <u>ratings</u> through the use of a scale, seeks to gain the maximum amount of information from each question. The latter approach requires only that the DM <u>rank</u> the different solution methods, giving a strictly comparative measure. Since a ranking of the four solution methods can be derived from a set of ratings under the first approach, the difference between the rating and the ranking assessment can be demonstrated. Consider Figure 5.1, on the following page, which compares these for the ZW method under Criterion 2 (ease of use).

The ratings show that subjects generally found the ZW method easy to use, with a mean rating of 7.5 (on a scale between 0 and 10). Using rankings, however, the ZW method had a mean rank of 2.7 , i.e., it was about the third hardest method to use on average. Comparatively speaking, then, ZW was quite

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1. Discussion with a member of the psychology department and a PhD student in psychology, along with a brief perusal of some texts indicates that no one approach tends to be favoured over the other.

hard to use, even though most subjects found it easy to use given the defined endpoints of the scale on which they assessed it.

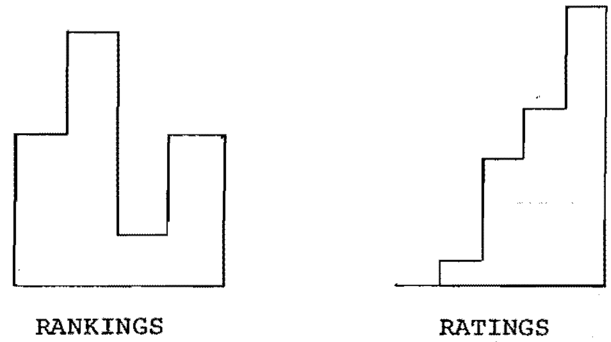


Figure 5.1 A Comparison of Ratings with Rankings

The DM's ratings under criterion 6 were also externalized using a line which has defined endpoints of "very hesitant to use" and "very willing to use". The DM is required to place four marks on this line, one for each method. This final questionnaire is also found in Appendix 3 (A3.5).

The CPU time and elapsed time (ET) are generated internally by the computer.

A considerable amount of qualitative information was also gathered from subject's comments etc.

5.3.2 Design Considerations

According to Cox (1958, p.5), the requirements for a good experiment include an absence of systematic error, sufficiently precise measurement, an experimental arrangement which is simple and that the uncertainty in the conclusions should be assessable. Most of the following discussion will be concerned with systematic error since the other factors are adequately catered for in the experimental design.

Since four solution methods are to be tested, it is necessary to have four different planning scenarios. If exactly the same MOLP was used for each method, it is probable that the DM would "lock in" to the solution obtained using the first method and thereby be biased in his or her assessment of all subsequent methods. A substantial "carry over" effect can be avoided when a different planning situation is used for each solution method. Also, systematic error can result if the four solution methods are always presented to the DM's in the same order, e.g., SWT is always the third method and always assessed under the optimistic planning situation. To avoid this situation, the order of the methods was randomized. The planning situations were, however, always presented in the same order, for it was considered that varying the order of these would only introduce further variability. This order was normal, pessimistic, optimistic and conservative.

However, some carry over effect is unavoidable. This will include the twin effects of increasing tiredness as the experiment progresses, and a learning effect as the subject becomes increasingly familiar with the case study situation. There will be some extent to which one effect will nullify the other.

The 24 subjects involved in the experiment consisted of graduate students and staff from the departments of Operations Research and Business Administration at the University of Canterbury. With one exception, all were familiar with LP models and the use of computer terminals. This is a somewhat biased sample when it is considered that it includes few people who have had considerable managerial experience in industry. Some consolation can be taken from Wallenius' (1975, p.1393) observation that "the data for the student and manager samples, (although) differing in several respects, seemed in many cases to be parallel".

5.3.3 The Experiment

All experimentation took place in the same room. Subjects were given a copy of the case study in advance to read at their own leisure. At the beginning of the experiment each subject was questioned to verify his or her understanding of the case study, and any questions were answered. A brief explanation of the MOLP was then given, with a distinction being made between efficient and inefficient solutions. It was made clear that each of the four solution methods dealt exclusively with efficient solutions. Before each method was actually used, a graphical representation of the method and how it worked was shown to each subject. Details of these prior explanations are contained in Appendix 3 (A3.3).

The subject then used the first solution method to "solve" the MOLP with the analyst providing additional, but impartial support as appropriate, e.g., answering questions and checking for incorrect responses to questions on the screen. Upon the completion of this first solution method, the subject's assessments of criteria 1,2 and 3 were taken, along with the internally generated CPU time and elapsed time. Each subject was encouraged to provide comments both during the method and upon completion. This procedure was then repeated for subsequent solution methods.

After all four solution methods had been used, the final question (criterion 6) was presented to the subject. Further opportunity was given for comments and discussion.

The average time taken for the experiment was about 90 minutes.

5.4 RESULTS

This discussion of the results will incorporate both the quantitative and qualitative data externalized from each subject. Since a number of criteria have been used to compare each solution method, the assessment of the four solution methods is itself a multiple objective problem. Criterion 6, which measures the relative preference for use in a practical decision making situation, should implicitly include the other criteria to a lesser or greater extent. This criterion can be thought of as an overall preference measure, i.e., a sort of utility function over the other five criteria, and it is probable that it will also include other factors not even considered in the experiment. Consequently, criterion 6 will accorded greater importance than the other criteria.

Because of the carry over effects mentioned previously, non-parametric statistics were used to analyse the data, rather than the standard ANOVA approach with t-tests. The Mann-Whitney U test and the Kruskal-Wallis oneway ANOVA were the main non parametric statistics to be used [see Kraft and van Eeden (1968) and Hull and Nie (1981) for details]. And since there are no reasonable grounds to a priori hypothesize that one method should be preferred to another for a given criterion, all tests are two tailed.

Results are presented in terms of both ratings and rankings, where the rankings have been derived from the ratings. Tables 5.1 and 5.2 (three pages following) provide a summary of the results. The raw data scores for each criterion are given in Appendix 3 (A3.6) with all the subjective measures scaled between Ø and 1Ø.

Further insight into scores for the four subjective criteria is provided in the frequency histograms of Figures 5.2 (ratings) and 5.3 (rankings) on the following two pages. These histograms are constructed such that the lowest ratings and worst rankings are on the left hand side of the histogram.

As expected, the rankings have a greater polarizing effect among the solution methods than the ratings, and in almost all instances confirm the results based on the ratings alone.

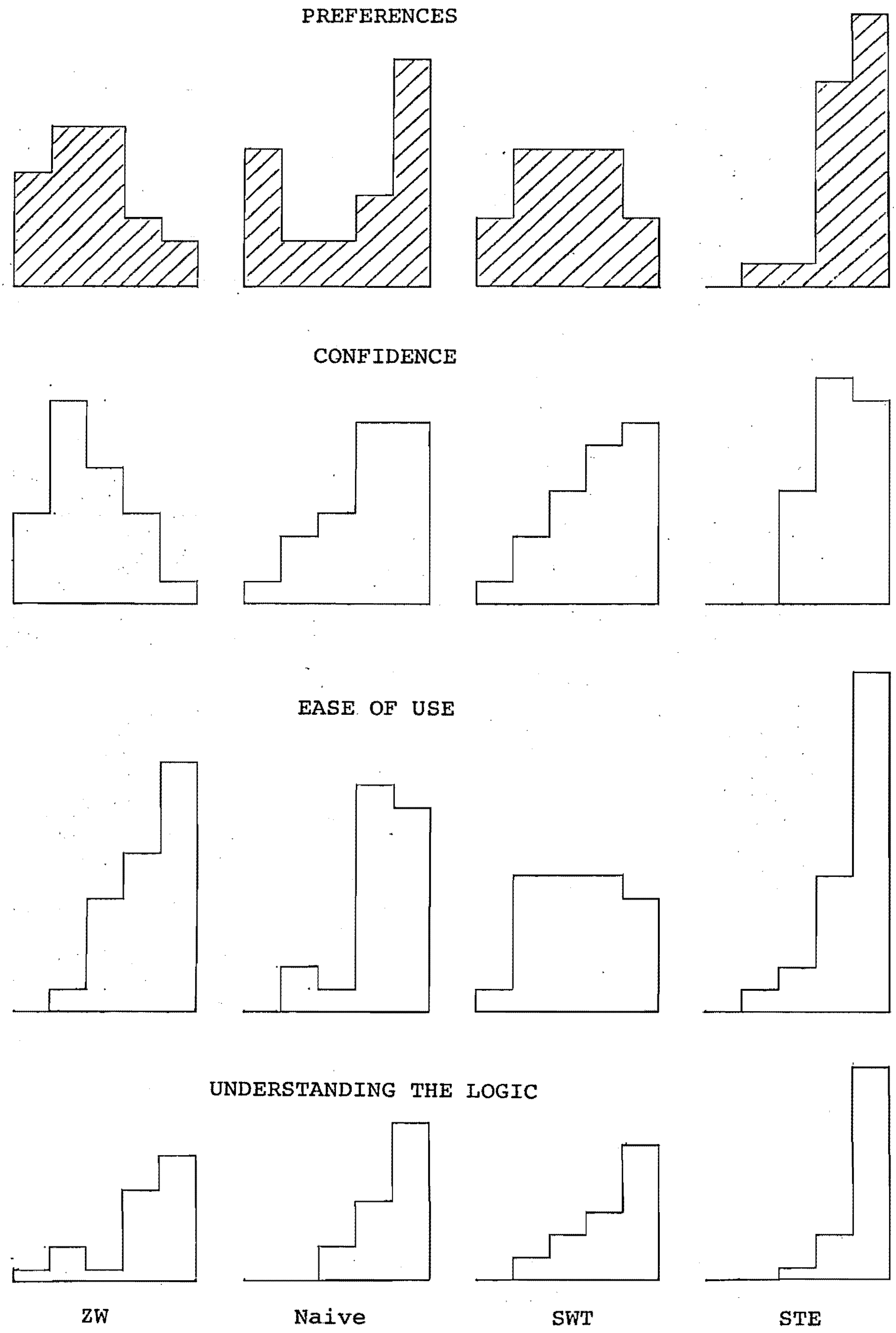


Figure 5.2 RATING Histograms for each Method

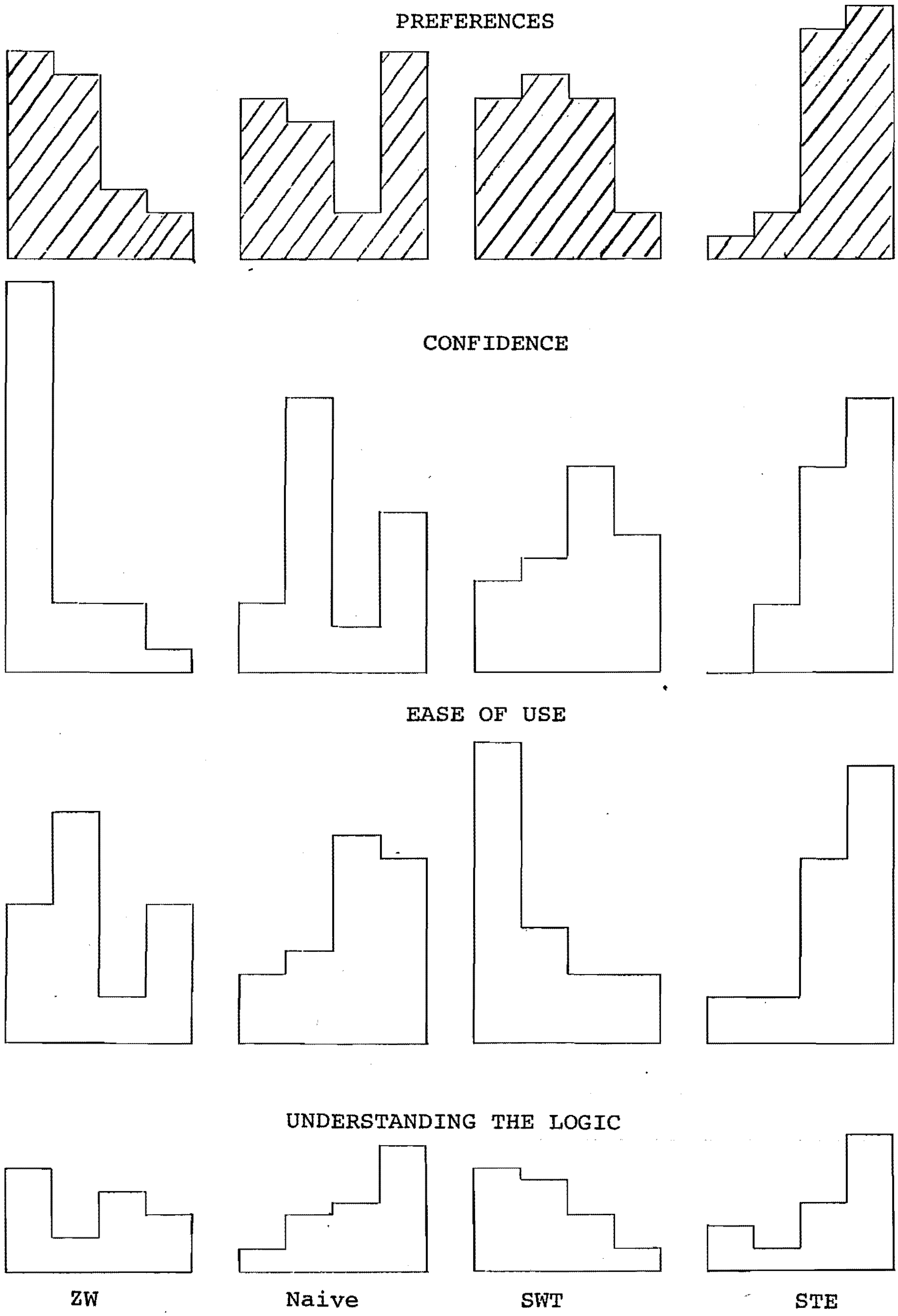


Figure 5.3 RANKING Histograms for each Method

MEDIAN RATINGS

	ZW	Naive	SWT	STE
6 Preference for use[a]	4.2	7.2	5.4	8.1*
1 Confidence[a]	3.4*	7.3	7.4	7.7
2 Ease of use[a]	8.0	8.0	5.5*	8.8
3 Understanding the logic[a]	7.9	8.6	7.7	9.0
4 CPU time (sec)	18.9	6.2*	17.0	63.7*
5 Elapsed time (min)	5.3*	10.3	18.0*	9.0

a - on a scale of 0 to 10.

* - significantly different from <u>all</u> other methods at 5% level of significance

Table 5.1 Median scores for each criterion

MEAN RANKINGS

	ZW	Naive	SWT	STE
6 Preference for use[a]	3.1	2.5	2.8	1.7*
1 Confidence[a]	3.5*	2.5	2.3	1.6*
2 Ease of use[a]	2.7	2.1[1]	3.2*	1.8[1]
3 Understanding the logic[a]	2.7	1.9[2]	3.0	1.9[2]

a - ranking using 1 (best) through to 4 (worst)

* - significantly different from <u>all</u> other methods at 5% level of significance.

Naive and STE significantly different from ZW and SWT at:

1 - 7% level 2 - 5% level

Table 5.2 Mean Rankings for the Four Subjective Criteria

5.4.1 Preference for Use

From Figure 5.2 it can be seen that the ZW method was the least preferred method, with 80% of the subjects scoring it below 6. This is in contrast to the STE method where 85% of the subjects gave it a score above 6. The distribution of scores for the Naive method is interesting in that it appears to be bimodal. Subjects tended to either strongly like or strongly dislike the method.

At a 5% level of significance, STE is preferred over the other three methods.

5.4.2 Confidence in the Method

Subjects had less confidence in the ZW method than any of the other solution methods; this result is significant at the 5% level. This relative lack of confidence can possibly be explained by examining the elapsed times for the solution methods. Subjects spent only a median time of 5.3 minutes on this method, compared with at least 9.0 minutes for the other methods. This indicates that subjects had little time to become familiar with this solution method; it is likely that additional experience with the ZW method may increase the level of confidence. It is not likely, however, that this lack of confidence stems from either the method being hard to use or difficult to understand, as shown in the rating histograms of Figure 5.2.

The other three methods all exhibited similar levels of confidence.

5.4.3 Ease of Use of the Method

The SWT method was considered by all subjects to be harder to use than any other method, at a 5% significance level. This is to be expected, since it is the only method which requires cardinal preference information. However, it should be noted that a method which is harder to use than others is not necessarily disadvantaged per se, since the greater amount of information provided may result in a "better" solution.

The statistical analysis indicates that STE and the Naive method were in general the easier solution methods to use.

5.4.4 Ease of Understanding the Logic of the Method

It is not unreasonable to expect the Naive method to perform well under this criterion, given the simplicity of the approach. In fact all methods performed reasonably well. Comparatively (i.e., using rankings), at the 5% level of significance, the logic of the SWT and ZW methods was found to be less clear than that of both STE and the Naive method.

With adequate familarization, it is expected that all these differences would be eliminated, with the logic of each method being understood clearly.

5.4.5 CPU Time

Under this criterion, the four solution methods differ significantly at the 5% level, with the exception of ZW and SWT. For a MOLP the size of the case study, these differences are unimportant. But for a larger, more realistic problem, they would become significant. Since Zionts and Wallenius (1982) have cited a number of large scale applications for their method and thereby demonstrated that the CPU time is not prohibitive for their method, only the STE method seems likely to be unusable (at least in its current form) for large MOLP problems.

5.4.6 Elapsed Time

Again the methods differ significantly at the 5% level of significance under this criterion. The exception is the Naive method and STE. Since a DM should be willing to invest more than 20 minutes in order to find an acceptable solution to any MODM which has more than insignificant consequences, it is not expected that these differences will have much effect in practical decision making situations. However, if a MODM requires repeated and frequent solution, then these differences will become apparent, and preference may well be toward the ZW method.

5.4.7 Additional Tests

In addition to these pairwise comparisons among methods, some further statistical analysis was carried out. This was to test for any systematic error that may be due to the position of the method, i.e., whether it was used first, second, etc. A two way analysis of variance [ANOVA, see Nie et al. (1975, Chapter 22)] was performed on the preference ratings using the two factors of method and position. The probability values at which the main effects of the method and position factors became significant were 0.000 and 0.120

respectively.

Further testing was done using the Mann-Whitney U test and the Kruskal-Wallis oneway ANOVA test, using the rankings. This was also to test for the effect of position on each subjective criterion, except that the test was performed on each solution method individually with the sample size reduced to six observations. Apart from the SWT method, where earlier assessments were generally more favourable than later ones (significant at 5% for criterion 2 only), there were no significant effects due to position. It was also found that for the criterion of elapsed time, position had a significant effect for the Naive method, but was not significant for the other solution methods. Table 5.3 below gives the median times of each position for the Naive method.

	Position			
	1st	2nd	3rd	4th
Median ET (min)	13.3	11.7	5.7	8.9

Table 5.3 Median Elapsed Times for Naive Method by Position

It is reasonable to expect the elapsed time to decrease as subjects gain greater familiarity with the decision situation. This is especially true for the Naive solution method which relies almost solely on the intuitive judgement of the DM and his or her "feel" for the problem in order to find a preferred solution. The low elapsed time for the third position is likely to be a consequence of random error resulting from a small sample size rather than other factors.

Overall, there seems to be a small effect due to position, as can be reasonably expected, but not to such an extent so as to invalidate the effect due to the different solution methods. Therefore this increases the confidence in the validity of the results as they apply to, and distinguish among the different solution methods.

The question was also asked whether or not the other five criteria were able to explain the variation in preferences for each method. Multiple linear regressions for each method showed that the other five criteria were unable to adequately explain this variation in preferences, at least in a linear fashion. A profitable direction for future research would be to seek to empirically define factors which do explain this variation in preferences for each individual solution method. This would provide substantial guidance as regards the future development of solution methodologies and their underlying principles.

5.4.8 Discussion and Implications

It is clear from the results that the STE method is preferred to the other three solution methods. This would indicate that the workings of this method are consistent with the decision making strategies of the majority of subjects.

Each method will be discussed in turn.

5.4.8.1 The ZW Method

This method fared worst under the four subjective criteria, but gave very acceptable values for CPU time and elapsed time. The general feeling of many of the subjects is described well by this single comment; "I understand the logic, it's easy to use, but I don't like it. I feel as if I have just purchased something from a fast talking salesman, and now I ask myself, 'Is that what I really wanted?'" It is almost as if the method is taking the responsibility of the decision from the DM. If this is so, i.e., that the DM does want to be able to say, "this is my solution", rather than feeling that it is the computer's solution, then users are likely to be prejudiced against the ZW method.

A number of subjects expressed a desire for greater flexibility; to be able to go back and change a previous choice. This behaviour was also evident in other methods. While a number of subjects agreed that they should be consistent in their decisions, they readily conceded that they were not, and many of them wanted this inconsistency to be accomodated by the solution method.

Also, some subjects found that choosing between two alternative solutions was often difficult. Three reasons were given. Firstly, that the two adjacent solutions were too close together to facilitate realistic comparison. Or secondly, they were too far apart, in which case a solution somewhere in between was usually preferred. This problem is likely to diminish as the size of the set of efficient extreme point solutions increases, and the results indicate that the ZW method was rated higher on those problems which had the greater number of extreme point solutions. The third reason why choices were often difficult was because both alternatives were considered to be "lousy" and subjects were not prepared to accept either of them. It seems that subjects much preferred to work with solutions that were acceptable to them.

Finally, it should be noted that despite the negative comments, the ZW method was not considered to be significantly different (at the 5% level) from SWT and the Naive method.

5.4.8.2 The Naive Method

Given its simplicity, this method performed surprisingly well. The bimodal nature of overall preference for the method can possibly be explained by contrasting those subjects who like substantial control (and therefore responsibility) with those who prefer that a method gives them more direction. One subject described the Naive method as being similar to fishing, in that there is a lot of luck as to whether a particular guess is feasible. The other view is that such an approach leaves plenty of room for intuition on the part of the DM.

Some subjects wanted the opportunity to constrain objectives at certain levels, rather than always having all three objectives changing proportionately from the desired solution to the achieved solution. This feature can easily be incorporated into the method, with the proportional mechanism operating among the remaining unconstrained objectives.

The Naive method tends to move quickly to the locality of the preferred solution, but the refining process of moving closer to the most preferred solution then tends to become somewhat haphazard. The Tradeoff (TO) method of Section 4.7.8 can provide local tradeoff information at each achieved solution, which should make this refining process easier. This difficulty in the Naive method of moving from a good solution to a better solution can possibly be overcome by using a hybrid solution method. Such an approach was suggested by four subjects and incorporated both STE and the Naive method. After some initial guesses to find a good solution with the Naive method, the STE method of using a contracting cone (centred around the current good solution) could then be invoked. Alternatively, the two methods could be used concurrently. This hybrid approach has the added advantage of incorporating two different decision strategies; namely the inherent incrementalizing strategy of the Naive method and the more global strategy of STE.

The log of previous solutions was, in general, found to be useful, with the subject able to build up a more global picture of the efficient surface after a number of iterations.

5.4.8.3 The SWT Method

It was expected that overall preference for this method will reflect both the preferences for assessing tradeoffs using worth values and the preferences for the particular way in which the method was presented to the subjects. If the method had been set up using the multiple regression approach to find indifference solutions, it is possible that the results would have been different. Given the general dislike for "black-box" type approaches, it is suspected that the ratings would have been lower for such an approach.

The method was harder to use than the other three solution methods and some subjects found it to be tedious. Assessment of tradeoffs was generally easy at the extremes, but more difficult for non-extreme values. Generally, subjects appreciated the opportunity to be confronted with their worth assessments via a graphical display, and to be able to edit and change their assessments.

It would be possible to reduce the CPU time by parametrically changing the RHS of the constrained objectives when generating the seven tradeoffs to be assessed at each iteration.

By comparison with the other three solution methods, SWT was the most "middle of the road" method in that extreme values did not dominate for any particular criterion. Colloquially speaking, it was neither loved nor hated.

5.4.8.4 The STE Method

The clear preference for this method would suggest that most subjects have a global approach to decision making; an approach which begins with an overview and continues to narrow down in the locality of the preferred solution. Such an approach would seem to have more intuitive appeal than the more incrementalist approach which starts at a single solution and then continues to move to a better solution at each iteration.

Under a global approach, the DM is in a much better position to form preferences, once he or she has an idea of what the outcomes are. For without a reasonably clear knowledge of the outcomes, the DM's goals are likely to be biased according to other less relevant historical information [Gimpl (1985)]. In other words, "you don't know what you want until you know what you can get". While the "best to worst" range information for the objectives (as given in the case study) gives some indication of the possible outcomes, it does not give any information on the extent to which the objectives move together. This can only be ascertained once a number of actual solutions have been presented to the DM.

Some subjects found it difficult to assimilate all the information contained in presenting six solutions over three objectives. With more than three objectives these difficulties would only be compounded. This suggests that there is a tension between the desirability for global information and the inability to process it. Further research is necessary to determine some structured ways to present this multidimensional information to a DM. A very simple approach would be to sort all solutions on one objective.

The major disadvantage with this method is the large amount of CPU time required, which would effectively render the solution method non-interactive for any large MODM. This may not be a disadvantage as is shown in the application described by Steuer and Schuler (1978), where the interaction was achieved by postal service. The method was interactive, but certainly not instantaneously so, and therefore hasty, and often inappropriate decisions should be eliminated. The CPU time can be reduced by solving less MOLP's at each iteration (e.g., nine instead of twelve). Also, after a few iterations when the cone is much smaller and the solutions much more similar, an updating procedure can be used, i.e., the second solution is found using the basis inverse of the first, and so on. Section 6.1.2.2 provides details of how an updating procedure can be implemented under the P1 formulation.

By narrowing down the cone of possible solutions at each iteration, the STE method does seek to converge to the most preferred solution. It is, however, a "forgiving method" in the sense that an inappropriate choice in the early iterations is not irrevocable. There is usually sufficient overlap between successive cones in these early iterations to allow the DM the flexibility to change direction. The method could be made even more flexible by allowing the parameter that controls the cone contraction to be changed at any iteration. Thus, if the DM wanted to backtrack, he or she could expand the cone around the current solution and continue from there. Such a facility would need to be used sparingly, lest the refining effect of the cone contractions be negated.

5.4.8.5 Presentation of Information

The presentation of information was also given high priority by the subjects. Many of them suggested the use of graphical displays, e.g., piecharts and bar graphs. It was also suggested that the actual numbers presented should be simplified, with costs being rounded to thousands of dollars and the percentages limited to only one decimal place. The manner in which information is presented is largely a cosmetic effect. It is not expected that presentation alone will greatly affect preferences among the different solution methods if all solution methods are making full use of the facilities available for presenting information. Comparatively poor presentation, however, is likely to significantly disadvantage a method.

5.5 CONCLUSIONS

The results of this experiment are consistent with the conclusions of Chapter 3. Subjects do prefer solution methods where they are in control, and where they are allowed to backtrack and change previous inconsistent decisions. Some "guiding influence" from the solution method was preferred to none, but not to the extent that the DM felt restricted. Perhaps this is the area of greatest tension; between flexibility or control for the DM and the extent to which the method aids the DM in the search for a preferred solution.

Also, preferences for using the different solution methods will in part be determined by the particular decision making environment. An environment in which frequent, quick analyses are common may well require different solution methods from the environment where there is an emphasis on strategic development and research.

Since as human DM's we are all different, there is no one best MODM solution method. It is likely that the ultimate solution method (if such a thing exists) will be a hybrid approach which can adequately accomodate the decision making strategies and behaviour of different decision makers.

CHAPTER 6 THE TRADEOFF METHOD AND THE SJT METHOD

6.0 INTRODUCTION

In this chapter two solution methods for MODM's are discussed. Although both methods can be used for non-linear MODM's, the emphasis will again be on linear models. The first solution method is the tradeoff method (TO); a brief outline of which was given in section 4.7.8, where it was shown how tradeoff information from the P1 formulation could be utilized as the basis for an interactive solution method. The second method is based on the concepts of Social Judgement Theory (SJT) which were discussed in some detail in Chapter 3. The SJT method has special application in the area of group decision making.

The motivation for these two methods derives from the conclusions of previous chapters, especially Chapter 3 as it dealt with the behavioural issues of decision making. The methods are an attempt to accommodate the often limited decision making ability of the human DM while making use of some of the theory of MODM's and especially the maxmin (P1) formulation.

The methods will be discussed under the following categories; motivation, methodological details, an example of the method being used and concluding with some possible extensions and practical considerations.

6.1 THE TRADEOFF METHOD (TO)

The TO method follows naturally from the maxmin formulation and the pairwise tradeoff information it provides. As outlined in Chapter 4, the method is designed to allow the DM to move over the efficient surface by using pairwise tradeoff information. Sections 4.6, 4.8 and 4.9 have already dealt with certain aspects of the TO method, and they will be referred to as necessary.

6.1.1 Motivation

The primary motivation behind this method is the concept of flexibility; to provide a mechanism which gives the DM freedom to move anywhere over the efficient surface and not be constrained by any prescriptive mechanism such as consistency of decisions with respect to a presumed utility function. As well as aspiring to be flexible, the TO method also seeks to provide information to aid the DM in moving from his or her current solution to a better or more preferred solution.

Also, the method takes account of man's inclination to "satisfice" (see Section 3.1.1), since it terminates when the DM is satisfied with the current solution. This avoids some of the "mystery" of other solution methods which often seem to stop abruptly, e.g., the method of Zionts and Wallenius.

Also, the TO method requires mainly ordinal information from the DM which will, at least compared with the provision of cardinal information such as marginal rates of substitution, reduce the burden of information provision on the DM. Furthermore, there is a significant difference between this method and the interactive solution methods where the DM provides his or her MRS at each iteration. In the TO method, rather than requiring the DM to provide his or her tradeoff information, the method provides its own tradeoff information to the DM. That is, instead of wanting to know the geometry of the DM's preferences, it tells the DM the geometry of the local efficient surface.

These two alternatives represent distinctly different methodological approaches. In the former, the DM provides the information which is analysed by the solution method, whereas in the latter the information is provided by the solution method and analysed by the DM. Both approaches are, at least in the theoretical sense, striving toward the same end; namely to find a solution at which the geometry of the feasible set is the same as the geometry of the DM's preferences. Some solution methods clearly fall into one of these two categories, whereas others are more a combination of the two approaches, e.g., the method of Zionts and Wallenius.

6.1.2 Methodological Details

The TO method may be regarded as an extension of the naive method, where additional information in the form of pairwise tradeoffs is provided at each iteration so that the DM can make a more informed guess at the next solution. The details of the method have already been outlined in Section 4.7.8 . Although this outline was for the simple two objective case, the concept of the method readily generalizes to any number of objectives.

Figure 6.1 below contains a possible set of pairwise tradeoffs for any two objectives f_i and f_j with all other objectives (f_k , k = 1,2,...,q , k <> i,j) held constant. Solution A is the achieved solution which resulted from a desired solution of $\underline{w}^o$. A tradeoff whereby f_i is increased may go beyond the current basis, i.e., beyond B to give $\underline{w}^1$ as the next desired solution. Solving P1 with weights of $\underline{w}^1$ will give the achieved solution $\underline{f}^1$, from which new pairwise tradeoff information can be presented to the DM for the process to continue.

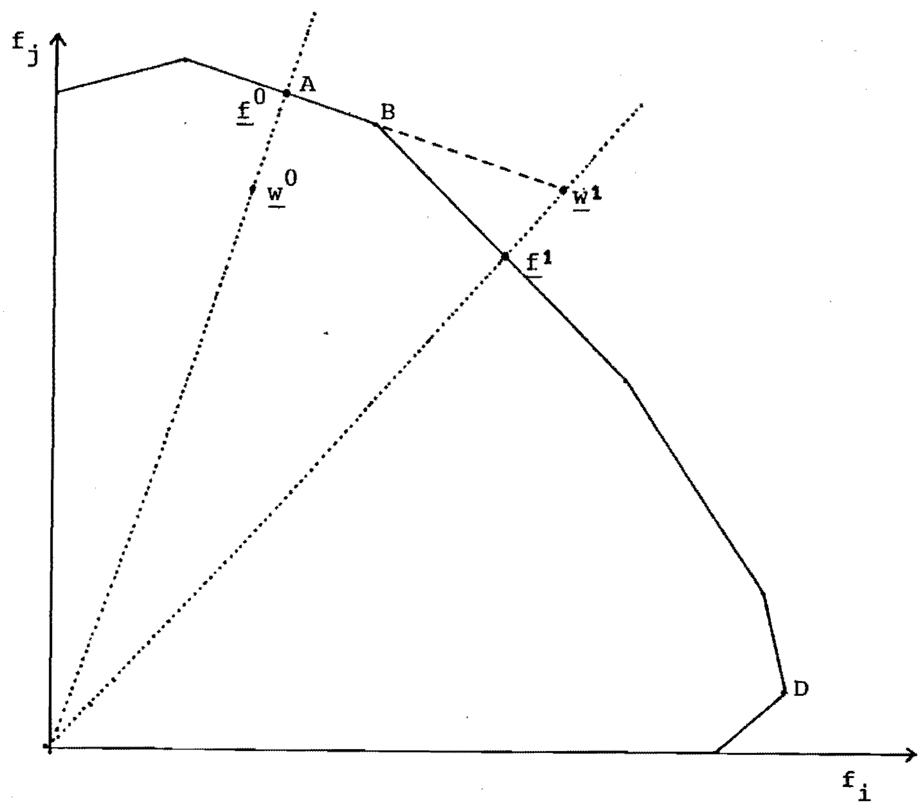
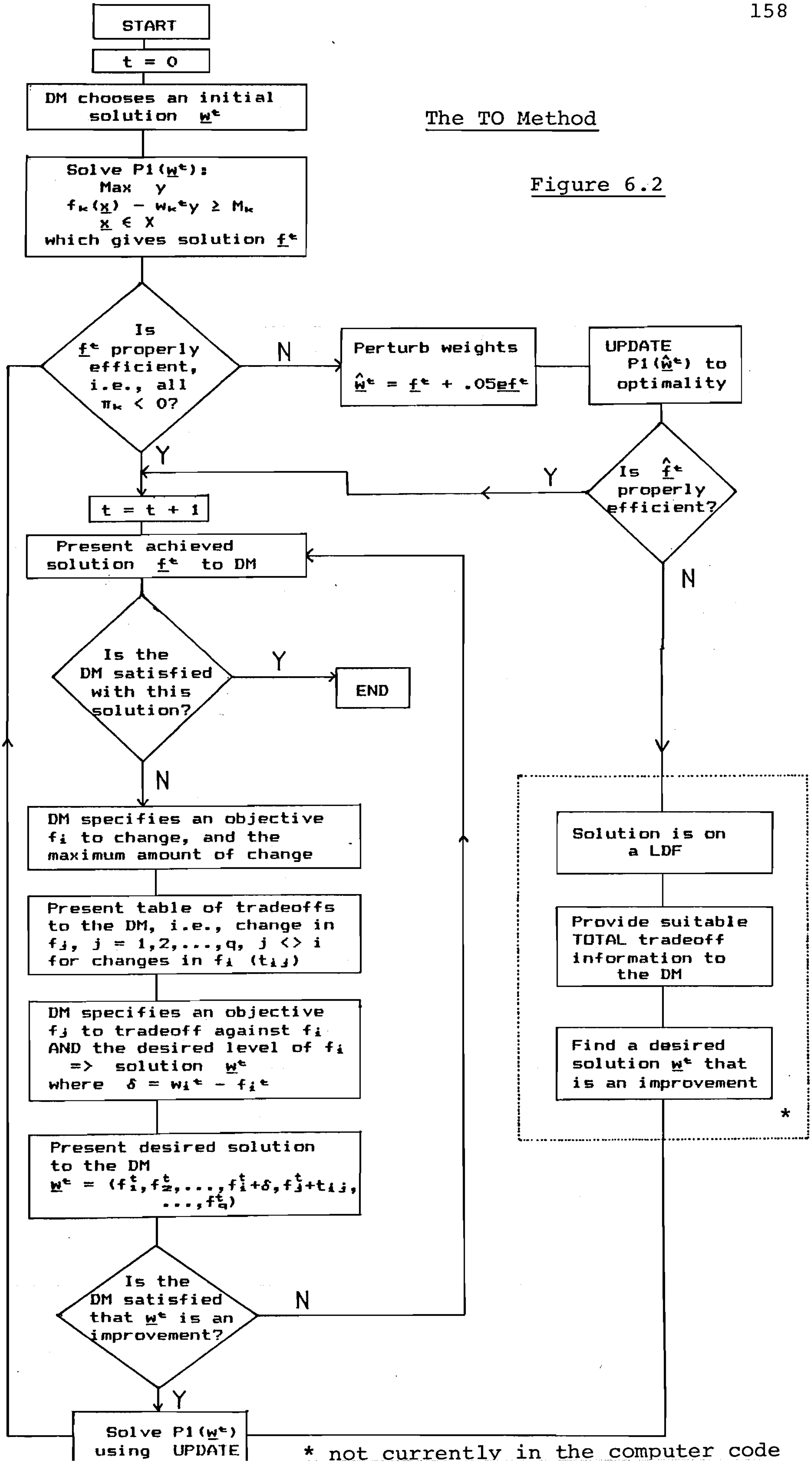


Figure 6.1

A flowchart of the TO method is detailed in Figure 6.2 on the following page.

The TO Method

Figure 6.2



* not currently in the computer code

The following discussion seeks to clearly explain the details of the flowchart.

6.1.2.1 Perturbing the weights (see also Section 4.9.2)

The reason for perturbing the weights is to ensure that the achieved solution is properly efficient; if it is not, then pairwise tradeoff information is not available since at least one objective constraint is not binding. Let $\underline{f}^t$ be an achieved solution which is not properly efficient and define weights

$$\underline{v}^t = (f_1{}^t, \ldots, f_s{}^t + 0.05, \ldots, f_q{}^t) = \underline{f}^t(1 + 0.05\underline{e})$$

where s is a subscript defined over all non-binding objective constraints and $\underline{e}$ is a vector with the value 1 for each non-binding objective constraint and zero otherwise. Solving $P1(\underline{v}^t)$ will give a solution just inside the efficient face, from which pairwise tradeoff information can be obtained.

As pointed out in Section 4.9.3, if "oscillations" occur, i.e., if as a result of perturbing the weights a previously binding objective constraint becomes non-binding, then the achieved solution is on a lesser dimensional face (LDF). Pairwise tradeoffs are not defined in this situation, which is addressed in more detail in Section 6.1.2.4.

6.1.2.2 Updating Procedure (UPDATE)

This procedure is only applicable for linear MODM's. It is designed to considerably reduce computing time by continuing from the optimal solution of the previous iteration, which in LP terminology is a "warm start". Such an approach is viable because changes in the achieved solution from one iteration to the next should not be large. The update procedure involves changing the y vector which will be basic at any properly efficient solution.

Assume that, at iteration t, the desired solution is $\underline{w}^t$. The corresponding y vector is

$$\underline{y}^t = (1, w_1^t, w_2^t, \ldots, w_q^t, \emptyset, \emptyset, \ldots, \emptyset)$$

where the 1 refers to the objective row and the zeros to the constraint set X. An updated form of $\underline{y}^t$ is found by multiplying by the optimal basis inverse at iteration t, B_t^{-1}.

$$\underline{y}^{new} = B_t^{-1} \underline{y}^t$$

An artificial pivot operation is then performed whereby $\underline{y}^{new}$ is pivoted in and the $\underline{y}^{old}$ vector which is in the optimal basis is pivoted out. As the resulting solution may be primally infeasible, a Phase I procedure or some equivalent is necessary. (In the author's code a single artificial vector is introduced to remove all infeasibilities and is later pivoted out by the Big M method (see Daellenbach and George (1978, pp.1Ø3-1Ø5) for an explanation of the Big M method.).)

To avoid cumulative numerical error, the P1 problem could be solved from the beginning or the optimal basis be reinverted after every fixed number of iterations, as is done in most linear programming codes. The UPDATE procedure has been included in the computer program of the TO method. In the example of Section 6.1.3 the CPU time was 2.3 seconds. The same example, <u>without</u> using the UPDATE procedure, took 4.3 seconds of CPU time, which represents a reduction of about 5Ø% . Savings will become even more obvious for larger

MOLP's.

6.1.2.3 Presentation of Pairwise Tradeoff Information

Under the P1 formulation, pairwise tradeoff information is available at every properly efficient solution. The question is how to make best use of this information, how can it be clearly presented to the DM? The approach taken for the TO method was to use the form of a table; although with hindsight, it would seem that some form of graphical presentation of the tradeoffs is preferred.

It would be possible to present all pairwise tradeoffs to the DM. This approach, then, would be the latter of the two discussed in Section 6.1.1, where the solution method presents all the information to the DM to be analysed. However, since it is unreasonable to expect the DM to be able to assimilate all the data contained in such a table of pairwise tradeoffs, a compromise approach has been adopted. Firstly, the DM chooses one objective that he or she desires to change; this will be deemed the reference objective, and is likely to differ from one iteration to the next. Then the DM specifies the maximum amount by which he or she would consider changing the reference objective. Based on this information provided by the DM, a table of the form (Figure 6.3 on the following page) can be presented to the DM.

This tradeoff table shows the values of each of the other objectives for different values of the reference objective, up to the maximum change specified by the DM. For example, if f_r is increased by an amount 2δ, then f_3 should decrease to f_3^2 with all other objective values remaining unchanged.

	Actual	1	2	3	max value
f_r	f_r^0	$f_r+\delta$	$f_r+2\delta$	$f_r+3\delta$	$f_r+4\delta$
f_1	f_1^0	f_1^1	f_1^2	f_1^3	f_1^4
f_2	f_2^0	f_2^1	f_2^2	...	
f_3	f_3^0	f_3^1	f_3^2	...	

Figure 6.3 A Tradeoff Table

However, the pairwise tradeoff information, which is contained in the Lagrange multipliers of the binding objective constraints, is only valid for the current basis. If the DM desires to make a tradeoff which goes beyond the current basis, then it is useful to have some approximation to the unknown efficient contour beyond this point. Otherwise the tradeoff table will consist of nothing more than a linear extrapolation of the pairwise tradeoffs at the current achieved solution. Attempting to approximate the unknown contour will result in the desired solution being closer to the achieved solution than if a simple linear extrapolation is used.

The intent of this section, therefore, is to find a good approximation to this unknown efficient contour.

Figure 6.4 (on the following page) illustrates possible pairwise tradeoffs for two objectives f_i and f_j with all other objectives, f_k , $k=1,2,...,q$, $k <> i,j$ held constant.

Solution A is the achieved solution. B can be calculated from the optimal basis inverse (Section 4.7.6). Solution D can be estimated from the matrix of extreme solutions. The unknown contour to be approximated, as represented by the fine dotted line, is a convex curve which lies between B and D. Any approximation to this contour will be convex, have a slope equal to the tradeoff ratio at A, and pass through both B and D.

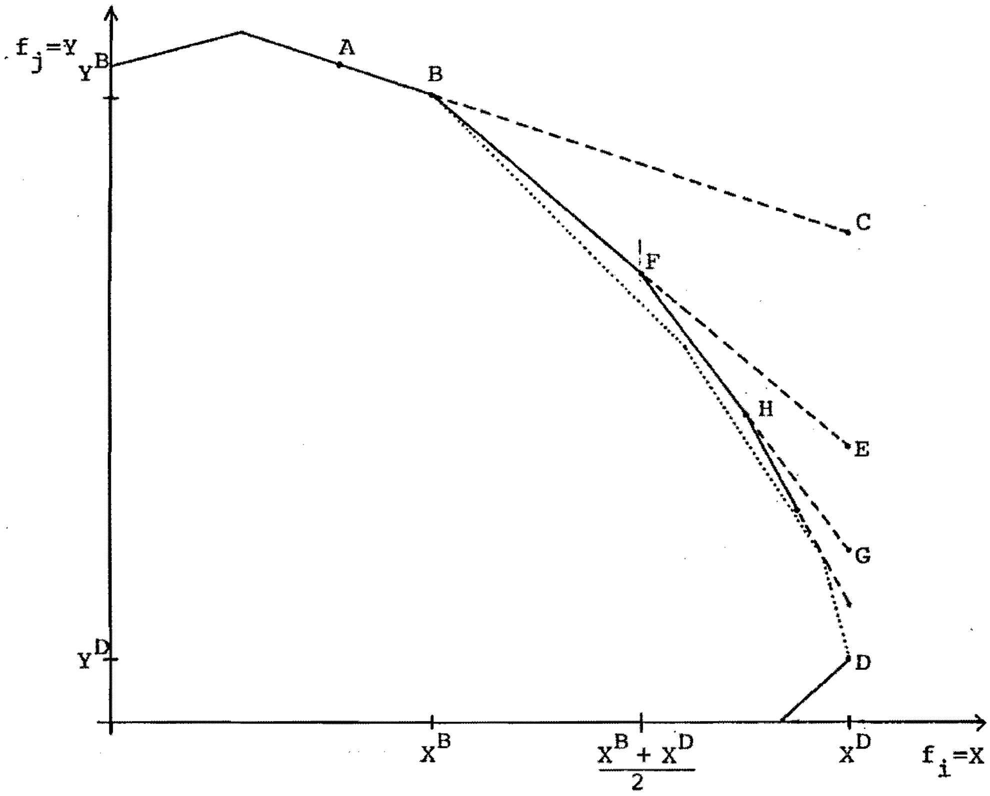


Figure 6.4

While both elliptical and cubic approximations to the unknown contour were tested, the most useful approximation was of the form of successive "halvings". Figure 6.4 illustrates this concept. The pairwise tradeoff is evaluated at A with t_A being the tradeoff ratio at this point. The approximation to the contour beyond B will lie somewhere between the line segments BC and BD. Since the contour is equally likely to be anywhere between BC and BD, the first approximation BE is calculated assuming that CE = ED. The equation of BE is given by

$$\text{Slope} = (Y^B - Y^D + t_A(X^B - X^D)) \;/\; 2(X^B - X^D)$$

$$\text{Intercept} = Y^B - X^B(\text{Slope}) \quad .$$

As an approximation, BE is likely to be more accurate in the range $[X^B,(X^B+X^D)/2]$. Therefore if the desired increase in f_i goes beyond point F in the diagram, then the halving process can be repeated beginning with a new approximation FG. An upper limit of 10 such "halvings" is suggested. The curve BFH represents part of the approximation to the actual, but unknown, contour.

Once this tradeoff approximation has been calculated for the desired increase in f_i, it is then presented to the DM in the form of a pairwise tradeoff table. In the TO method the increase in f_i has been split into four equal steps in order to give the DM a better appreciation for the tradeoffs. The use and presentation of the tradeoff table is clearly illustrated in Section 6.1.3 .

6.1.2.4 Solutions on a Lesser Dimensional Face (LDF)

If the achieved solution is on a LDF then it is not possible to provide pairwise tradeoff information. Consequently, such a situation necessitates the use of total tradeoffs which would be presented in terms of as few objectives as possible. This also will require a change in thinking for the DM; in order to move over the efficient surface he or she will have to think in terms of total tradeoffs, rather than pairwise tradeoffs.

Total tradeoff information is not so readily available as was the pairwise information. However, at the LDF solution, the ratios of the dual variables of the binding objective constraints are valid and do give the change in one objective for a unit change in another. But for any tradeoff that is made, the values of all objectives corresponding to the non-binding constraints will also change, and neither the amount nor the direction of the change can be ascertained from the current LDF solution using the P1 formulation. Section 6.1.4.3 demonstrates how suitable tradeoff information can be provided in this situation by using a more general formulation.

Without incorporating an extension for the LDF, the TO method in its current form is not suitable for MODM's having a large number of LDF's. However, in the author's experience to date, the occurence of the LDF situation has been found to be relatively uncommon among the MOLP models which have been used for research purposes. Also, since a LDF cannot occur in a MODM which has only two objectives, the TO method is ideally suited to such models. Certainly in terms of realistic application, at least in the New Zealand context, models with more than two objectives are unlikely to have much acceptance at this point in time [Read (1985)].

6.1.3 Example of Use

The following pages (Figure 6.5) provide an example of the TO method as used for the normal planning situation of the case study. Each screen is presented as it would be seen by the DM at the computer terminal. Costs have been rounded to thousands of dollars in order to facilitate presentation.

```
RUN
#?
#RUNNING 1796
Enter name of data file (AUG__) plus fullstop,
AUGNM/ZW.

Enter initial guess at most preferred solution [ % ]
  COST
100
  STKOUT
100
  LABOUR
100

Enter ICASE  1-Actual  2-Normalized
1
```

```
          I  PREVIOUS GUESS        I  RESULTING SOLUTION
          I  Actual   Normalized % I  Actual   Normalized %
----------I------------------------I------------------------
*COST     I  1538.07        0.0    I 1648.54          44.3
*STKOUT   I     6.36        0.0    I   12.34          44.3
*LABOUR   I     0.00        0.0    I    6.73          44.3

Do you want to STOP with this solution? Enter Y or N
N

Enter name of objective you would most like to change
COST

Enter maximum amount of change; + implies improve, - worsen
30
```

```
Actual tradeoff values for COST

           Actual      Step1      Step2      Step3      Step4
        I           I
 COST   I 1648.54   I 1641.04    1633.54    1626.04    1618.54
--------I-----------I------------------------------------------
 STKOUT I   12.34   I   13.12      13.62      14.11      14.60
        I           I
 LABOUR I    6.73   I    7.18       7.64       8.10       8.56
        I           I

Choose the objective to tradeoff against COST
STKOUT

Choose the desired level of COST
1620
```

```
          I  CURRENT SOLUTION        I    DESIRED  SOLUTION
          I  Actual   Normalized % I   Actual   Normalized %
----------I--------------------------I-------------------------
*COST     I  1648.54          44.3   I  1620.00          32.9
*STKOUT   I    12.34          44.3   I    14.50          60.4
*LABOUR   I     6.73          44.3   I     6.73          44.3

Is this desired soln. an improvement? Enter Y or N
N

There are two options, please choose one of them.
  1.Choose another objective to change instead of COST
  2.Choose another objective to tradeoff against COST
    or choose a different amount to tradeoff
2
```

```
Actual tradeoff values for COST

            Actual       Step1      Step2      Step3      Step4
         I            I
 COST    I 1648.54    I 1641.04    1633.54    1626.04    1618.54
---------I------------I---------------------------------------------
 STKOUT  I    12.34   I    13.12      13.62      14.11      14.60
         I            I
 LABOUR  I     6.73   I     7.18       7.64       8.10       8.56
         I            I

Choose the objective to tradeoff against COST
STKOUT

Choose the desired level of COST
1628
```

```
          I  CURRENT SOLUTION        I    DESIRED  SOLUTION
          I  Actual   Normalized % I   Actual   Normalized %
----------I--------------------------I-------------------------
*COST     I  1648.54          44.3   I  1628.00          36.1
*STKOUT   I    12.34          44.3   I    13.98          56.5
*LABOUR   I     6.73          44.3   I     6.73          44.3

Is this desired soln. an improvement? Enter Y or N
Y
```

```
          I      PREVIOUS GUESS      I   RESULTING SOLUTION
          I  Actual   Normalized % I  Actual    Normalized %
----------I------------------------I-------------------------
*COST     I  1628.00        36.1    I  1634.60        38.7
*STKOUT   I    13.98        56.5    I    14.22        58.3
*LABOUR   I     6.73        44.3    I     7.08        46.6

Do you want to STOP with this solution? Enter Y or N
N

Enter name of objective you would most like to change
LABOUR

Enter maximum amount of change; + implies improve, - worsen
2
```

```
Actual tradeoff values for LABOUR

           Actual     Step1      Step2      Step3      Step4
        I          I
 LABOUR I     7.08 I    6.58       6.08       5.58       5.08
--------I----------I-----------------------------------------
 COST   I  1634.60 I 1642.80    1651.00    1659.96    1671.40
        I          I
 STKOUT I    14.22 I   15.03      15.39      15.76      16.13
        I          I

Choose the objective to tradeoff against LABOUR
COST

Choose the desired level of LABOUR
6
```

```
        I  CURRENT SOLUTION        I     DESIRED  SOLUTION
        I  Actual   Normalized %  I   Actual     Normalized %
--------I-------------------------I--------------------------
*COST   I  1634.60       38.7     I  1652.23          45.8
*STKOUT I    14.22       58.3     I    14.22          58.3
*LABOUR I     7.08       46.6     I     6.00          39.6

Is this desired soln. an improvement? Enter Y or N
Y
```

```
          I  PREVIOUS GUESS         I   RESULTING SOLUTION
          I  Actual   Normalized %  I   Actual   Normalized %
----------I-------------------------I-------------------------
*COST     I  1652.23        45.8    I  1652.23          45.8
*STKOUT   I    14.22        58.3    I    14.22          58.3
*LABOUR   I     6.00        39.6    I     6.00          39.6

Do you want to STOP with this solution? Enter Y or N
N

Enter name of objective you would most like to change
STKOUT

Enter maximum amount of change; + implies improve, - worsen
1.5
```

```
Actual tradeoff values for STKOUT

             Actual      Step1      Step2      Step3      Step4
         I          I
 STKOUT  I    14.22 I    13.85      13.47      13.10      12.72
---------I----------I------------------------------------------
 COST    I  1652.23 I  1653.83    1655.42    1657.02    1659.32
         I          I
 LABOUR  I     6.00 I     6.10       6.19       6.29       6.44
         I          I

Choose the objective to tradeoff against STKOUT
LABOUR

Choose the desired level of STKOUT
13
```

```
         I  CURRENT SOLUTION        I   DESIRED  SOLUTION
         I  Actual   Normalized %   I   Actual   Normalized %
---------I--------------------------I------------ ------------
*COST    I  1652.23        45.8     I  1652.23          45.8
*STKOUT  I    14.22        58.3     I    13.00          49.2
*LABOUR  I     6.00        39.6     I     6.32          41.6

Is this desired soln. an improvement? Enter Y or N
Y
```

```
          I  PREVIOUS GUESS           I   RESULTING SOLUTION
          I  Actual   Normalized %    I  Actual     Normalized %
----------I---------------------------I--------------------------
*COST     I  1652.23          45.8    I  1652.23           45.8
*STKOUT   I    13.00          49.2    I    13.00           49.2
*LABOUR   I     6.32          41.6    I     6.32           41.6

Do you want to STOP with this solution? Enter Y or N
Y

#ET=6:12.6 PT=2.3 IO=0.6
```

Figure 6.5 Example of the TO Method

6.1.4 Discussion and Possible Extensions

6.1.4.1 User Experience

Unfortunately the TO method was not in an operable state when the experiment of Chapter 5 was set up and run. However some of the subjects who were involved with the experiment have also had experience with the TO method. Although their assessments for this method were not a part of the original experiment and therefore will have little statistical validity, their average preference rating was favourable. Specifically, for a sample size of 9, the median rating for willingness to use was 7.8 which compares favourably with the other four solution methods tested in Chapter 5.

Some users demonstrated a desire to avoid incrementalizing, i.e., moving over the efficient surface in small steps. Instead there was a tendency to opt for large tradeoffs in order to move exactly to the desired value of a given objective in a single iteration; a more "quantum" approach to decision making. Since the validity of the tradeoff approximation is likely to diminish as the size of the tradeoff increases, the TO method is better suited to an incremental approach to decision making [e.g., Lindblom (1959)].

Also, the presentation of pairwise tradeoff information in the form of the tradeoff table previously described, is helpful when there are more than three objectives. For a MOLP with six objectives, the tradeoff table immediately enabled the DM to get a "feel" for which objectives were significantly affected when the reference objective was changed. Those objectives which do not change much can then be excluded from any decision strategy of the DM, thus simplifying the decision making process at that iteration.

6.1.4.2 Presentation of Information

The cosmetics of presentation will have an effect with regard to the desirability of the TO method (as seen by the DM). Presentation can be further enhanced by providing graphical displays where appropriate (such as in the tradeoff table) and maintaining a log of all past solutions so that the DM has a record of where he or she has been and can go back and "start again" from any one of these previous solutions.

It is also possible to present the information contained in the tradeoff table as marginal changes rather than absolute values. Certainly, the option can be provided so that both forms of presentation are available. However, changes such as these are only cosmetic and do not change the underlying principles of the method.

The TO method can be further enhanced by identifying when the edge of the efficient set is reached; a point beyond which pairwise tradeoffs are invalid. For example, referring to Figure 4.9 in Chapter 4, it would be unrealistic to allow a pairwise tradeoff (increasing f_1 and decreasing f_2) beyond edge CA. This will ensure that the DM is less likely to move off the efficient surface through the use of an unrealistic pairwise tradeoff. However, while this may increase the accuracy of the tradeoff table, it may also restrict the range of tradeoffs available.

As a final point regarding presentation, it should be made clear to the DM that the information in the tradeoff table is effectively only a guess at what the resulting solution will be when one objective is changed. The tradeoff table does not purport to be able to give exactly the solution which results from a given tradeoff; it is only an estimate. It was found that once users were accustomed to this, small discrepancies between the desired and achieved solutions were tolerated.

6.1.4.3 The Reference Point Approach - Resolving the LDF Situation

The research undertaken at IIASA by Wierzbicki and his colleagues on the reference point approach[1] demonstrates a number of similarities with the TO method. Their work also provides for an approach which deals with the LDF situation. Both the TO method and the reference point approach effectively use a scalarizing function where the weights are determined by the reference point or the desired solution specified by the DM. Each provide a vector of dual variables at the optimal solution from which tradeoff information can be derived. The TO method endeavours to use this information specifically in the form of pairwise tradeoffs which are used by the DM to move over the efficient surface from one solution to the next. In the explanation of the reference point approach (Lewandowski and Grauer (1982)) it is not specified exactly how this dual vector is used, except that the DM uses it and any other available information to determine another reference point.

In the reference point approach a somewhat different scalarizing function is used from that of minimizing the maximum achievement as in the P1 formulation. The scalarizing function is defined as

$$s(\underline{w}) = -\beta \min w_k - \underline{ew} \qquad [1]$$

where β is a coefficient greater than or equal to q and $e = (e_1, e_2, \ldots, e_q)$ is a non-negative vector of parameters. $\underline{w} = (\underline{f}-\bar{\underline{f}})$ where $\bar{\underline{f}}$ is the reference point specified by the DM. Minimizing this scalarizing function gives rise to the following formulation.

1. This work at IIASA on the reference point approach only came to the attention of the author in April 1985, through a visitor in the department, Dr B. Murtagh. This was after the the TO method had been developed.

$$\text{RP}(\underline{y},\underline{w},\underline{e}) : \quad \text{Min} \quad \underline{y} - \underline{ew}$$
$$E\underline{y} + D\underline{w} \le \emptyset \qquad [2]$$
$$-\underline{w} + C\underline{x} = \bar{\underline{f}}$$
$$\underline{x} \in X$$

where $\underline{y}$ is an auxiliary variable and D and E are appropriate vectors and matrices. At an optimal solution to [2], the dual vector $\underline{\pi}$ to the constraints $-\underline{w} + C\underline{x} = \bar{\underline{f}}$ defines a hyperplane $H = \{\underline{f} : \underline{\pi}(\bar{\underline{f}}-\underline{f}) = \emptyset\}$. This hyperplane gives the equation of the portion of the efficient surface on which the current solution lies, from which pairwise or total tradeoff information can be made available to the DM.

If this formulation is used then the difficulty of providing total tradeoff information to the DM in the situation of a LDF is overcome.

6.1.4.4 Extensions in Principle

The literature review of Chapter 3 and the experiment of Chapter 5 suggest some further extensions to the method; in this case extensions to the underlying principles. The tradeoff approach ideally accommodates compensatory strategies, but makes no provision for any non-compensatory strategy that the DM may wish to pursue. A non-compensatory strategy can be included by allowing the DM to constrain objectives at certain levels. Practically this can be achieved by replacing the relevant objective constraints in the P1 formulation with an upper bound on each of the objectives. This has the added advantage of reducing the number of objectives which would then be examined via a compensatory strategy, i.e., tradeoffs. Chapter 7 deals in more detail some approaches for reducing the dimensionality (in terms of objectives) of the MODM.

A further extension would be to add a facility not unlike the method STE as detailed in the previous chapter. At any iteration the DM would be able to get an "explosion" of points around the current solution by simply specifying the number of points required and the size of the cone within which they are generated. If the cone size is reasonably small, the additional computational time can also be kept to a minimum by using the UPDATE procedure.

The result of these extensions would be more a suite of methods or a package, rather than just one particular method of solution. DM's who have a decision making strategy which is primarily a compensatory one are likely to favour the TO portion of the package with some recourse to the other facilities; while others may prefer to use the STE type approach with some objectives constrained. Based on the properties of the P1 formulation, all these facilities can easily be made available, thus providing a flexible solution tool which can accommodate the DM in whatever his or her decision making strategy may be. The package would provide a sound analytical base with the only optimization being to locate efficient solutions from among the entire set of feasible solutions. The DM can make use of this analytical base as required and still leave sufficient room for more intuitive strategies.

6.2 THE SJT METHOD

This method consists of two distinct parts. First, the judgement policy of a DM is captured. And secondly, this information is then used to find a preferred solution. Also, unlike many other solution methods, the SJT method is directly applicable to the multiple decision maker situation.

6.2.1 Motivation

This method was born out of a desire to make use of the skills of two disciplines; Psychology with its insight into decision making behaviour and Operations Research with its ability both to represent the relevant aspects of a decision problem by a suitable mathematical model and to solve it. The need for an understanding of decision making behaviour (i.e., judgement processes) is a direct consequence of using multiple objective models. This is because subjective preference information is required from the DM before a final and satisfactory solution to the model can be found. However, in contrast, subjective information is unnecessary where there is only a single objective in the decision model (in the absence of alternative optima).

As outlined in Chapter 3, Social Judgement Theory (SJT) is primarily concerned with the description of human judgement processes. The underlying postulate of the theory is that the uncertain nature of human judgements is a consequence of the uncertain and generally ambiguous environment within which such decisions are made. This situation was first modelled by Brunswik in his Lens Model [e.g., Hammond et al.(1975)], with the actual method for capturing the judgement policy of the DM being based on a regression model. Once the DM's policy has been captured, this information can then be utilized to solve the MODM and thereby obtain a satisfactory or preferred solution. It is not expected that this will be the final most preferred solution; rather it is to be viewed as a good starting point.

Mumpower et al.(1979) describe well the spirit of this approach. They speak of a Symmetrical Linkage System which is an analytic approach for linking social values with scientific information. It is symmetrical since equal attention is given both to social values and scientific information. This approach has been applied to the Denver regional air pollution problem where possible scenarios were defined with their resulting outcomes being derived from a simulation model. SJT was used to locate a small set of preferred scenarios and a small set of preferred outcomes. Some further interaction with the simulation model resulted in a few outcomes which reflected the preferences of the Denver residents.

The case of the single DM will be considered first and then extended to multiple decision makers.

6.2.2 A Single Decision Maker

6.2.2.1 Methodology

The procedure can be outlined in five steps.

1. About 15-20 efficient solutions are generated as being representative of the entire set of efficient solutions for the MODM.

2. Each solution is presented to the DM in terms of the levels of achievement of each objective. This can be either using actual values and/or some commensurable scale such as fractional achievement.

3. The DM provides a rating for each solution using a 20 point cardinal scale.

4. The judgement policy of the DM is captured using a multivariate regression model, e.g.,

$$\text{rate} = w_1 f_1 + w_2 f_2 + \ldots + w_q f_q + \text{error}$$

5. This judgement policy is used as the objective in the MODM in order to find a preferred solution.

Steps 1 and 2 allow the DM to become familiar with the range of outcomes of the decision problem, after which he or she is in a much better position to be able to form goals or preferences. These preferences will be reflected in the ratings of Step 3. And in order for the DM to form some realistic preferences, it is important that the solutions or outcomes presented to the DM are <u>representative</u> of the entire set of efficient solutions. To achieve this a large number of random solutions are generated within the bounds (U_k and M_k, $k = 1,2,\ldots,q$) as estimated from the matrix of extreme solutions. These solutions are then filtered to find a subset of the most dissimilar ones. Each of these solutions is then used as weights in the P1 formulation which transforms each set of weights into an efficient solution. This is an almost identical approach to that used in the STE method (Sections 2.3.2.4 and 5.2.4).

Step 4 captures the judgement policy of the DM using a linear regression model. The resulting regression equation is not presumed to be the DM's utility function, since the intent of SJT is simply to describe. It is, as pointed out in Section 3.3.2, a paramorphic representation of judgement in that it simulates well the outcomes or ratings of a given policy, but is unlikely to reflect the actual decision processes. Therefore, the fact that a linear regression model is used does not presuppose that a DM must also have a linear utility function, or for that matter have any form of utility function.

And it is not even necessary to use a linear regression equation. Mumpower et al.(1979) suggest the following more general form of regression equation.

$$\text{rate} = \sum_{k=1}^{q} \beta_k \left[a(f_k-\bar{f}_k)^2 + bf_k + c \right] + \text{error} \qquad [3]$$

where $\bar{f}_k$ is the mean of f_k over all observations. This more general form allows for non-linear relationships between the rate and each f_k , $k=1,2,\ldots,q$.

Also, it is important to distinguish between the actual regression coefficients $\underline{w} = (w_1,w_2,\ldots,w_q)$ and the beta weights $\underline{\beta} = (\beta_1,\beta_2,\ldots,\beta_q)$. The beta weights are used to define the actual judgement policy of the DM and are the regression coefficients of the standardized or normalized solution values.

The main purpose of externalizing the DM's judgement policy is to remove the decision making from outcome or solution space into policy space. While both are necessarily of the same dimension, the latter should (depending on the aptness of the model) have much of the error due to inconsistency in judgement removed from it.

Finally, in Step 5, the actual regression coefficients $\underline{w}$ are used as weights to form a single composite objective function,

$$\sum_{k=1}^{q} w_k \left(\sum_{j=1}^{n} c_{kj} x_j \right).$$

Optimizing the MODM with this objective will give a solution which reflects the judgement policy (i.e., relative preferences) of the DM. This solution will be an efficient extreme point solution for any linear MODM. And as previously mentioned, this solution is only intended as a good starting solution; therefore some further local searching may be necessary in order to find a satisfactory solution. A possible approach would be to present the DM with a small explosion of points around this starting solution and iterate in a similar manner to the STE method. Alternatively, the TO method could be used from this starting point.

6.2.2.2 Predictive Ability of the Linear Regression Model

The first reason for using a linear regression model is a pragmatic one; a non-linear model would result in a non-linear optimization problem, which is more difficult to solve. However, this becomes less of a consideration if the MODM is already non-linear in the constraint set. The second reason comes from the discussion of Chapter 3 which indicated that the linear model is very robust and often able to outperform non-linear models in terms of predictive ability [e.g., Dawes and Corrigan (1974), Schoemaker and Waid (1982)].

The predictive ability of a linear regression model has been briefly tested using the data of the normal planning situation from the case study of Chapter 5. Appendix 4 lists all 55 efficient extreme point solutions to the MOLP under the normal planning situation. Two different types of utility functions were used to examine the predictive ability of the linear regression model; non-linear utility functions and "fuzzy" utility functions. The fuzzy utility function is defined as

$$U_{fuzz} = U(f_1, f_2, \ldots, f_q) + \alpha z \qquad [4]$$

where $U = a_1 f_1 + a_2 f_2 + \ldots + a_q f_q$, i.e., the true linear underlying utility function

z is a normal random deviate with mean of zero and standard deviation of one

α is a factor which controls the extent of the inconsistency

The fuzzy utility function is used in order to simulate the behaviour of an inconsistent decision maker.

The performance of the SJT method under a fuzzy utility function was examined as follows. Initially, a true linear utility function U was chosen, and the solution of maximum utility $\underline{f}^o$ was found from the set of extreme point solutions. Then the five steps of the SJT method were performed using ratings provided by the fuzzy utility function. The resulting solution $\underline{f}^1$ was then compared with the solution of maximum utility, $\underline{f}^o$.

For example, U was defined by

$$U = -[\ 4.0E\text{-}4(f_1 - 1740797) + 2.0f_2 + 6.0f_3\ ]$$

which gives $\underline{f}^o$ = (\$1,614,330 11.77% 8.97%)

with $U(\underline{f}^o) = -26.77$

Then 16 randomly generated plans (also based on the normal planning situation) were evaluated using the fuzzy utility function. These plans can also be found in Appendix 4. A value for α of 5 was used in the fuzzy utility function. When compared with the ratings for $\alpha = 0$, i.e., a perfectly consistent DM, the standard deviation of the ratings increased by about about 70% on average. 20 different sets of ratings with their resulting solutions $\underline{f}^{1j}$, j = 1,2,...,20 were then compared with $\underline{f}^o$. The results are shown in the histogram of Figure 6.6 below where the x axis measures the ordinal position of the solutions $\underline{f}^{1j}$, j = 1,2,...,20 when compared with the solution of maximum utility $\underline{f}^o$.

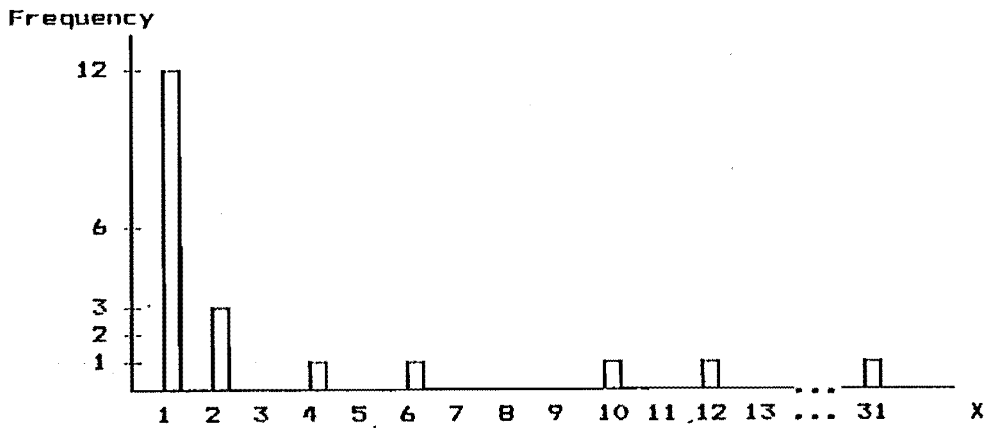


Figure 6.6

This analysis supports the claims for the robustness of the linear regression model, with the solution of maximum utility found over 50% of the time. This was despite the considerable inconsistencies in the execution of the judgement policy, as represented by the fuzzy utility function.

There is, however, another factor which the regression model has no control over; namely the shape of the efficient surface. If, for example, there are only a small number of extreme points then the chances of the regression model finding the extreme solution of maximum utility are much higher; except that, unless the DM has a linear utility function, the solution is much less likely to be satisfactory.

Experimentation with non-linear utility functions showed the linear regression model to be less adept at finding the extreme point solution of maximum utility. It did, however, perform surprisingly well when a standard multiplicative form was used. Specifically the form was

$$(d_1+1)\ (d_2+1)\ (d_3+1)$$

where d_k is the range norm

$$d_k = (f_k-M_k)/(U_k-M_k)\ ,\ k=1,2,3\ .$$

In this situation the extreme solution which had the third highest utility value was found.

6.2.2.3 Negative Regression Coefficients

One practical issue to be considered is when some of the regression coefficients are negative. In theory this would not be expected to occur, since preferences are assumed to be monotonic in the sense that more is always preferred to less. However in practice, this can occur due to inconsistency on the part of the DM or if the linear form of the regression model is inappropriate. If negative weights are used in forming the composite objective function, there is no guarantee that the resulting solution will be efficient. Some modification is therefore necessary before forming the composite objective function.

Assume that the regression coefficients w_j, $j = 1,2,\ldots,p$ (with $p < q$) are negative. The simplest approach is to set each w_j, $j = 1,2,\ldots,p$ equal to zero before calculating the composite objective function. Alternatively, the beta weights can be modified as follows. Let

$$\beta_m = \max_{1 \le j \le q} \{ \mathrm{Abs}(\beta_j) \} \qquad [5]$$

and define $\hat{\underline{\beta}} = \underline{\beta} + \underline{e}\beta_m$ where $\underline{e} = (1,1,\ldots,1)$ is 1xq. $\hat{\underline{\beta}}$ consists only of positive beta weights. These new beta weights can then be transformed back into the actual regression coefficients by

$$\hat{w}_k = \hat{\beta}_k(\sigma_R/\sigma_{fk}) \qquad [6]$$

where σ_R is the standard deviation of the ratings
and σ_{fk} is the standard deviation of the solution values for each objective, $k = 1,2,\ldots,q$.

And since σ_R is constant, it can be excluded from the transformation as it is only the <u>relative</u> proportions of the regression coefficients which are important in determining the resulting solution.

6.2.2.4 Example of Use

In practice it is envisaged that about 20 solutions or plans, as shown in Figure 6.7 below would be presented to the DM in a hard copy form, i.e., on paper, rather than being assessed interactively at a computer terminal.

```
    PLANNING MODEL for XYZ Co. Ltd  -  Normal Situation

PLAN 1
           Percentage achievement of objectives
      0-------------------50--------------------100  VALUE

COST   #################                          $1,635,291

STKOUT###############################                 13.71%

LABOUR#######################                          7.17%

Enter rating on next line ( 0 - 20 )
28.2
```

Figure 6.7

A hard copy of the plans to be rated is chosen because it enables the DM to assess the plans at his or her leisure. Also, with a hard copy, it is easier to go back and change earlier ratings if necessary.

From these ratings a policy can be derived. To simplify presentation the beta weights are normalized so that they sum to one. A statistical analysis is performed to assess whether or not the policy weights differ from zero. If the DM is satisfied with the policy, a solution is found;

otherwise the ratings (or possibly the policy) are modified. Figure 6.8 below provides an example of policy analysis.

```
                              POLICY ANALYSIS

 Policy has a predictability measure of 0.874

 Relative weight profile
       0--------------------0.5-------------------1.0  Weight

COST   *******************                              0.43

STKOUT *****                                            0.15

LABOUR ******************                               0.42

       0--------------------0.5-------------------1.0

 Significance test of policy weights at 5% level

   All policy weights are significant

Are you satisfied with this policy?
Y
```

```
      PLANNING MODEL for XYZ Co. Ltd  -  Normal Situation

                  Resulting Management Plan

 I----------------------------------------------------------I
 I       COST            STKOUT             LABOUR          I
 I      dollars        av % short         % laid off        I
 I----------------------------------------------------------I
 I                                                          I
 I    $1,563,533         11.77%             8.97%           I
 I                                                          I
 I                                                          I
 I----------------------------------------------------------I
```

<u>Figure 6.8</u>

6.2.3 Multiple Decision Makers

The simplest case of only two decision makers will be discussed in this section, although the extension to three or more DM's is not difficult [e.g., Hammond et al.(1975)]. It is to be expected that there will be disagreement between two DM's; the contribution of SJT is to aid conflict resolution by breaking the disagreement down into two categories. These categories are disagreement due to differing policies and disagreement which is simply due to inconsistent execution of those policies. Obviously the method does not propose to be able to make two DM's completely agree; it is rather a mechanism or aid to help resolve disagreement by transferring the decision making from outcome space into policy space.

6.2.3.1 Methodology and an Example

The steps of the method for the multiple DM situation follow that for the single DM up to and including Step 4 where the individual policies of each DM are captured using a regression model.

Again an example will be used to illustrate the proposed method in action using the data of the normal planning situation. The two DM's were simulated using two fuzzy utility functions, where both had the same underlying utility functions, i.e.,

$$U^1 = U^2 = -[\ 4.0E\text{-}4(f_1 - 1740797) + 2.0f_2 + 6.0f_3\ ]$$

with values for α of 5 for U^1 and 3 for U^2. Also, the same set of 16 plans (as used to simulate the single DM) were again used for each DM. The overall correlation between the two sets of ratings was Ø.5949 which is indicative of a moderate amount of disagreement. However the two policies are almost identical, with the correlation between the predicted values from each policy being Ø.9297 . The two policies are detailed in Figure 6.9 below.

```
               POLICY  ANALYSIS

Policy 1 has a predictability measure of 0.718
Policy 2 has a predictability measure of 0.874

                       Relative weight profile
       0--------------------0.5--------------------1.0  Weight

COST   *******************                              0.436
COST   ###################                              0.431

STKOUT *****                                            0.136
STKOUT ######                                           0.154

LABOUR ******************                               0.428
LABOUR ##################                               0.415

       0--------------------0.5--------------------1.0

 Significance test of policy weights at 5% level

 All policy weights are significant
```

<u>Figure 6.9</u>

The Lens Model equation of Section 3.4, which is expressed entirely in terms of correlations, can be applied to these results. In this context and in the absence of nonlinearities, the model can be stated as

overall agreement	=	policy agreement	X	consistency of execution

i.e., $$r_a = GR_1R_2 + C[\ (1-R_1^2)(1-R_2^2)\ ]^{(1/2)}$$

where C measures the correlation of the residuals. The actual values are

$$\begin{aligned} 0.5949 &= (0.9297)(0.7182)(0.8743) + (0.0328)(0.1141)^{(1/2)} \\ &= 0.5838 + 0.0111\ . \end{aligned}$$

The Lens Model equation shows clearly how the disagreement on the outcomes (ratings) is due almost entirely to judgemental inconsistency with a very small residual factor. This is not unexpected since both DM's had the same underlying utility function; this analysis simply confirms this, and demonstrates the predictive ability of the linear model despite considerable judgemental inconsistency.

Obviously this is the more desirable situation, with virtually all disagreement being a result of inconsistency. A more realistic situation, however, is illustrated by the following example where two fuzzy utility functions are again used, but with <u>different</u> underlying utility functions.

$$\underline{U}^1 = -[\ 4.0E-4(f_1-1740797) + 2.0f_2 + 6.0f_3\ ]$$
$$\underline{U}^2 = -[\ 5.0E-4(f_1-1740797) + 4.0f_2 + 6.0f_3\ ]$$

Taken individually, Steps 1 to 5 of the SJT method give these two extreme solutions, for each of the two fuzzy utility functions. The value for α was 3 .

$$\underline{f}^1 = (\ \$1,614,330 \quad 11.77\% \quad 8.97\%\ )$$
$$\underline{f}^2 = (\ \$1,592,750 \quad 8.72\% \quad 12.25\%\ )$$

Using the same data set of 16 plans derived from the normal planning situation, the following policies result (as shown in Figure 6.10 below).

```
               POLICY  ANALYSIS

  Policy 1 has a predictability measure of 0.874
  Policy 2 has a predictability measure of 0.979

 Relative weight profile
       0-------------------0.5------------------1.0  Weight

COST   *******************                            0.43
COST   ###################                            0.43

STKOUT *****                                          0.15
STKOUT ##########                                     0.25

LABOUR ******************                             0.42
LABOUR ##############                                 0.32

       0-------------------0.5------------------1.0

 Significance test of policy weights at 5% level

   All policy weights are significant
```

Figure 6.10

Examining the correlations in the Lens Model equation reveals that outcome disagreement is almost entirely due to differing policies since $r_a \approx G$.

$$r_a = GR_1R_2 + C[\,(1-R_1^2)(1-R_2^2)\,]^{(1/2)}$$

$$0.3387 = (.3893)(.8743)(.9787) + (0.0560)(0.0099)^{(1/2)}$$

$$= 0.3331 + 0.0056$$

Having now made this information available to each DM, it is assumed that the DM's can use it and come up with a policy upon which they can agree. Such an analysis is likely to show which objectives are the cause of the policy disagreement. Assume that the combined, agreed upon policy is

$$\hat{\underline{\beta}} = (0.43 \quad 0.22 \quad 0.35) .$$

This can be transformed into the weights $\underline{w}$ used in the composite objective function as detailed in [6]. The resulting solution is

$$\hat{\underline{f}} = (\$1,569,514 \quad 11.77\% \quad 12.25\%)$$

The above examples serve to illustrate how the SJT method operates, both with single and multiple DM's. Even if there is some hesitation as regards Step 5 of the method, i.e., using the regression coefficients to find an extreme solution, the method still has considerable merit. The actual process of externalizing the preferences of the DM's by capturing the judgement policies will bring much insight to the decision making process. This procedure of confronting the DM(s) with their actual assessments has already proven to be useful, as in the SWT method of Chapter 5, where after the DM has assessed seven pairwise tradeoffs these are shown to him or her in graphical form. Such an analysis makes no implications as to a correct manner of assessment, it only presents the information to the DM in a form that is easy to assimilate.

SJT presents the ratings of the DM(s) both in policy space and in outcome space. Policy space has the advantage of having stripped the ratings of a large amount of judgemental inconsistency, with the result that the issues which are at the heart of the disagreement should become clear to both parties. While most of the SJT method has focussed on this externalization of preferences into policy space, outcome space information can also be valuable. SJT speaks of functional forms, which are simply a two dimensional graph of the ratings against the values of a single objective. An example is given in Figure 6.11 below.

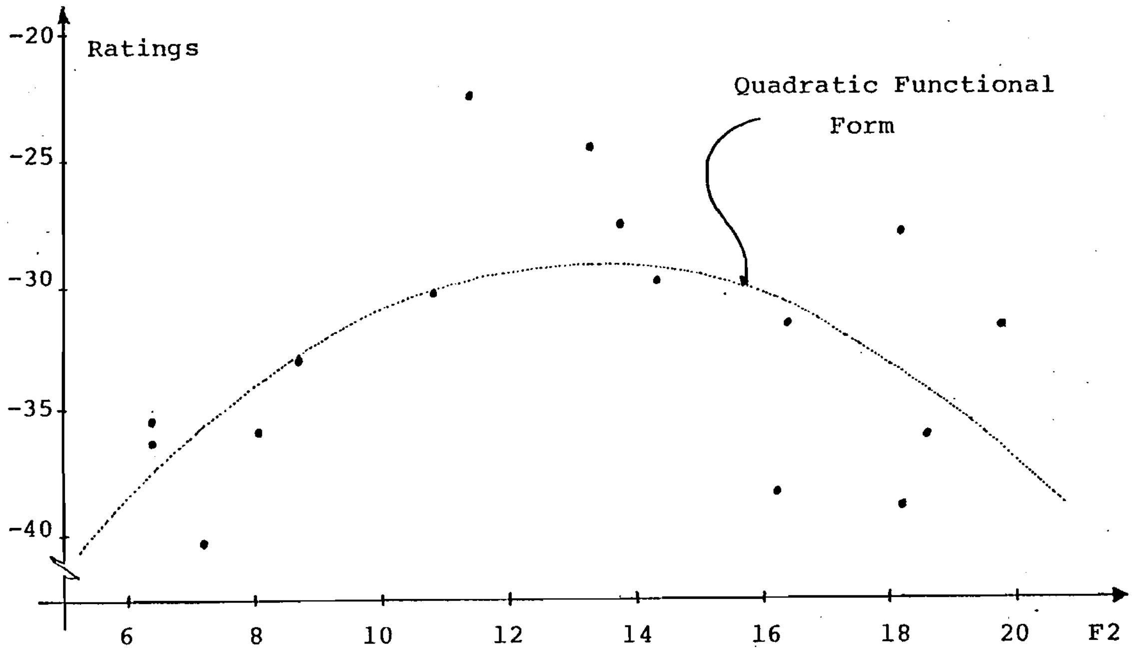


Figure 6.11

Again the 16 randomly generated plans from the normal planning situation were used along with a fuzzy utility function where $\alpha = 3$.

As mentioned at the beginning of Section 6.2, it is the intent of SJT to describe. And this simple process of confronting the DM(s) with their decisions (i.e., describing them) will greatly aid the formation of realistic preferences, while also providing a better understanding into actual judgement strategies. This is the main intent of the SJT method, however a means of using this externalized preference information to find a preferred solution has also been given.

6.3 CONCLUSION

This chapter has presented two different approaches for solving the MODM. Both have come from an investigation of the properties of the P1 formulation and also from a desire to accommodate and aid the DM in his or her decision making. While neither method has been subjected to substantial testing in a practical sense, the work that has been carried out, especially with regard to the TO method, has indicated that both the solution methods are indeed practicable.

CHAPTER 7 REDUCING THE NUMBER OF OBJECTIVES

7.Ø INTRODUCTION

This chapter examines the situation where the dimensionality of the objective space is to be reduced. It deals with the somewhat special situation where this reduction is performed a posteriori, i.e., after the MODM has been formulated.

Traditionally, and especially in the context of linear programming, the approach was to first formulate the decision problem so that there is only a single objective to be optimized. This involved either combining the various objectives into a single composite objective [e.g., Churchman et al.(1957, Chapter 2)] or including all but one objective as constraints in the initial formulation. (Notice how well Soland's characterization of an efficient solution (Section 2.1.2) encapsulates these two different approaches.) With the advent of multiple objective models, the emphasis has generally been to seek to keep the number of objectives at a minimum as the MODM is being formulated. The process of reducing the number of objectives is effectively performed before or during formulation.

In contrast, this chapter addresses the other extreme, i.e., where the dimensionality of the objective space needs to be reduced after formulation, in order to facilitate realistic evaluation of the decision situation. The motivation for this came initially from considering applications of MOLP to land use planning, where there are often a large number of objectives to be considered. For example, Martinson (1977) considers 39 objectives in a land use model for the BLM, Colorado; Jameson (1984) has 26 objectives in a recreational planning model for the Central North Island of New Zealand; and Austin and Cocks (1978), in a land use model for the South Coast of NSW, Australia, have 36 policy objectives. As the literature of Chapter 3 clearly indicates, it is unlikely that any DM will have the ability to effectively process such large amounts of information as contained in the above examples.

The emphasis in this chapter will be on analytic approaches for reducing the number of objectives in a MODM, rather than on approaches which are more subjectively oriented. A subjective approach is one whereby objectives are either included or excluded on the basis of preference information which is provided by the DM. Any successful approach for reducing the number of objectives will be a balance between both analytic and subjective approaches. This balance is well illustrated in Keeney and Raiffa (1976, p.35) where they cite examination of the literature, analytical study and casual empiricism as being relevant considerations in determining the objectives for a MODM. Keeney and Raiffa (p.43) provide a good example of this third consideration with Ellis' test of importance. This test is a subjective approach where the DM is asked if he or she feels that the best course of action would be altered if a particular objective is excluded. A positive response to this question would therefore suggest that the objective should be included in the MODM.

7.1 LITERATURE

In their second chapter on the structuring of objectives, Keeney and Raiffa (1976) devote considerable attention to the specification of objectives for a decision problem. They consider hierarchical objective structures and also the desirable properties of a set of objectives. Among these desirable properties are that the set of objectives should be complete, operational and of minimum size. Only this final property of minimum size will be considered in this chapter.

As a first step in problem formulation, Churchman et al. (1957) describe an "editing" procedure for reducing the number of objectives. They give two criteria for eliminating objectives; the first concerns elimination of objectives which are simply means of attaining other objectives. Statistically, such objectives would exhibit high positive correlation. The second criterion is to eliminate objectives which seem to be relatively unchanged for different solutions in decision space. This can be measured statistically by the variance.

In the case of commensurable objectives, Olenik and Haimes (1979) have used a hierarchical decomposition approach in an attempt to reduce dimensionality. In their approach, commensurable lower level objectives are aggregated into a single higher level objective. They provide an example where different recreational pursuits, such as swimming, hunting and picnicking, can be aggregated into a single recreation objective. Solving the higher level problem first then provides a framework for solutions at the lower level.

Also, there are a number of approaches which, although not directly aimed at reducing the number of objectives, do provide useful concepts. Steuer and Harris (1980) describe a filtering process for choosing a subset of the most dissimilar extreme points from some larger set, where each extreme point is measured across q different objectives. This approach has been used in the STE method (see Sections 2.3.2.4 and 5.2.4). The measure of dissimilarity is based on a distance metric of the form

$$\sum_{k=1}^{q} (a_{ki} - a_{kj})^2 \qquad [1]$$

for each pair of extreme points i and j and where a_{ki} is the value of objective k at efficient solution i. This approach could be changed to measure dissimilarity of objectives across a number of extreme points.

A similar approach which uses a different measure of dissimilarity has been adopted by Starr and Greenwood (1977). They use an entropy measure for controlling the generation of dissimilar solution alternatives. Entropy is a measure of the complexity of a system (i.e., the amount of information it carries). In their approach, additional alternatives continue to be added to the system until the marginal increase in entropy reaches some prespecified level. Zeleny (1974) has also used an entropy measure for calculating the relative importance of objectives. His analysis is based on the assumption that the relative importance of an objective is directly related to the average amount of information conveyed by that objective, with respect to a given set of efficient solutions. While Zeleny used this approach to weight objectives rather than to reduce the size of the objective set, the latter is an obvious extension.

Also, a number of dimension reducing statistical techniques are found in the literature. These include principal components analysis, factor analysis and multidimensional scaling.

7.2 DATA SOURCE

Any analytical approach will require a suitable data source on which to base the analysis.

The most appropriate data source for any dimension reducing analysis will be some subset of the set of efficient solutions. This is because the set of efficient solutions will reflect how the objectives "move" relative to each other within the actual constraint set of the MODM. However, practical considerations will tend to determine the actual size and availability of this data set. The matrix of extreme solutions would be the smallest practical data source and, in effect, provides bounds for for the entire efficient set. Alternatively, (for a linear MODM) a more ideal data source would be the set of all efficient extreme point solutions. Realistically, however, the actual data source will be somewhere in between. Perhaps the simplest way of deriving a suitable data source is again to use the approach of the STE method, based on the P1 (or P2) formulation. In this approach a set of randomly generated weights is filtered to find a subset of dissimilar weights, each of which are then used in the appropriate formulation to give a set of efficient solutions.

Before beginning any analysis, the data set to be used should be standardized so that each objective has a mean of zero and a standard deviation of unity. This will facilitate realistic comparison. Objectives which have a relatively small variance could be eliminated at this stage. This would use the second criterion of Churchman et al., i.e., the value of the objective is relatively unchanged over a range of solutions.

7.3 APPROACHES FOR REDUCING OBJECTIVE DIMENSIONALITY

The aim of any dimension reducing approach is to find a subset of objectives which adequately characterize the decision problem. Hopefully, the size of this subset will be sufficiently small (and still meaningful) so as to simplify the decision making task of the DM.

Before considering any analytical approach for reducing objective dimensionality, it is important to again stress the balance between subjective and analytic approaches. Although an objective may be eliminated for analytical reasons, the DM may desire that particular objective to be retained. The purpose of reducing the number of objectives is to make the DM's decision making task easier; it is not to confound the DM by removing objectives which are meaningful to him or her. It is therefore assumed that the approaches to be mentioned below will always be carried out with the active participation of the DM.

Before possible approaches for reducing objective dimensionality are examined, two different measures of dimensionality will be discussed.

7.3.1 The Statistical Measure

The statistical measure of dimensionality aims to find the minimum number of dimensions which explain the maximum amount of variance. The appropriate statistical method is principal components analysis which finds those orthogonal components that can explain most of the variation. There are, however, two difficulties with the method of principal components. Since the orthogonal components derived by the analysis are linear combinations of the original objectives, the objectives lose their original identity. It is likely that a component which is a linear combination of non commensurable objectives will be meaningless.

The second difficulty is simply a feature of principal components analysis. Generally the first few components explain about 90% of the variability; consequently, even a large number of objectives is likely to be reduced down to two or three principal components.

There is also a third difficulty which is associated with interpreting the correlation matrix (on which principal components analysis is based), rather than with the actual performance of principal components analysis itself. Obviously, if objectives exhibit high positive correlation, this is indicative of some redundancy and, in this case, the number of objectives can be reduced. But negative correlation, even though it also implies the existence of some linear dependence, may not be an indication that objectives should be eliminated. Consider the objectives in a MODM, and the concept of tradeoffs; by virtue of being convex, the tradeoffs should exhibit negative correlation.

In any MODM it would be reasonable to expect that a number of objectives will be negatively correlated. Statistical methods, however, will tend to exclude linearly dependent objectives, regardless of whether the correlation is positive or negative.

Therefore, if the correlation matrix is used, then only the subset of most orthogonal objectives will be retained. This could possibly eliminate the very tradeoffs that the DM was interested in examining when the problem was formulated. However, there will also be instances where negative correlation does represent genuine redundancy; thus the resolution of this difficulty will ultimately depend on the DM's understanding of the objectives.

The difficulties associated with using a correlation matrix of objectives can possibly be overcome by using a "pseudo-correlation matrix" which has the desirable properties of values near unity for high positive correlation and values close to zero for high negative correlation. Zero correlation would be approximately halfway in between. Some form of distance metric suggests itself. The metric described below has such properties, and is only one of a number that would suffice.

$$d_{kj} = [(\sum_{i=1}^{r} 4-\text{abs}(a_{ki}-a_{ji}))^2 / 4q]^{(1/2)} - 1 \qquad [2]$$

where a_{ki} is the value of objective k at efficient solution i, for $k = 1,2,\ldots,q$ and $i = 1,2,\ldots,r$.

The resulting distance matrix $[d_{kj}]$ has a diagonal of unity, with all other values less than unity.

The determinant of the correlation matrix gives a measure of the amount of linear independence among the objectives. Consequently, as the determinant becomes larger, the variability within a particular set of objectives also increases. While this is a useful measure of dimensionality, it should be noted that it can only be used for comparing subsets of objectives that are of the same size. For example, it is valid to compare combinations of objectives of size 3 or of size 4, but not to compare between those of size 3 and 4.

This determinant measure can be based either on the original correlation matrix or on an appropriately defined distance matrix (e.g., as described in [2]).

7.3.2 The Entropy Measure

The statistical approach seeks to find the subset of objectives which maximizes variability. Alternatively, an entropy measure can be applied to find that subset of objectives which maximizes variety. That is, it can be used to find that subset of objectives which is the most complex or contains the most information.

The definition of cumulative entropy, as given in Starr and Greenwood (1977), provides a suitable measure. It is defined as

$$H = -(q/\ln[q-1]) \sum_{k=1}^{q} \sum_{j=1}^{q} (d_{kj}/D_k)(\ln[d_{kj}/D_k]) \qquad [3]$$

where $D_k = \text{Max } d_{kj}$ and

$$d_{kj} = [\sum_{i=1}^{q} (a_{ki}-a_{ji})^2]^{(1/2)}$$

As can be seen from the definition, this measure of entropy is based on a simple unweighted euclidean distance metric. And like the statistical measure based on the determinant, this entropy measure is only valid for comparing subsets of the same size.

7.3.3 Dimension Reducing Approaches

From the previous discussion, at least two measures for reducing dimensionality are apparent. On the basis of these measures, some approaches for reducing dimensionality suggest themselves. One possibility is a sequential building type approach and another is the "reverse" of it, i.e., an elimination approach which begins with every objective included. There are obvious parallels with stepwise regression, especially the methods of backward elimination and forward selection [see Draper and Smith (1966, Chapter 6)]. A third approach which in concept lies somewhere between these two will also be discussed.

7.3.3.1 Forward selection and backward elimination

In the forward selection approach, objectives are added one at a time until sufficient are included to maximize variability (or variety, depending on the measure used). Two criteria are necessary for this approach. The first is a criterion for determining the order in which objectives are selected and the second is for determining termination. Under the statistical measure the approach could be as follows. Begin by choosing one objective which definitely must be included (as specified by the DM). Then form all possible pairs with that objective and calculate the determinant of the resulting correlation matrix, which will be of size 2x2. Choose the pair which has the highest determinant and continue by forming all triples with that pair and every other objective. Termination is found subjectively, i.e., when the DM considers that sufficient objectives have been included.

If the entropy measure is used instead of the statistical measure, then the termination criterion is much more obvious; it occurs when the increase in entropy from adding an additional objective is less than some prespecified tolerance level.

The backward elimination approach is effectively the reverse of the forward selection approach. This approach begins with all objectives initially included, and seeks to successively eliminate objectives until a satisfactory subset of objectives is obtained.

7.3.3.3 A third approach

The previous two approaches are quite rigid in that only the unique set of objectives which scores highest on a given measure of dimensionality is considered. The author's experience with the three different measures (i.e., cumulative entropy and the determinant of the correlation and distance matrices) has indicated that the three measures perform well in discriminating between obviously "bad" and obviously "good" combinations of objectives. But their discriminating ability diminishes considerably among sets of objectives which are all reasonably "good". Consequently, choosing only the highest scoring combination of objectives at each stage may be inappropriate.

This third approach, then, is perhaps a more pragmatic one in that it incorporates experience with the discriminatory performance of the different measures. The approach is described below. Initially include all objectives and examine the resulting distance and correlation matrices. From this choose the number of objectives required to be in the reduced subset of objectives. For example, assume that there are 25 objectives and a workable subset of 8 is desired. It is obviously not practical to assess the approximately 1 million combinations of 8 objectives. Instead, randomly generate about 100 different sets of 8 objectives and apply either a statistical or entropy measure to each. Since each set will then have a score associated with it, choose the 10-15 sets which have the highest scores. For these sets of 8 objectives, calculate the frequency with which each objective appears. This frequency will indicate the relative importance of each objective in contributing to the total score. Then choose the 8 objectives which occur with the highest frequency and use this as a starting solution.

7.4 PRACTICAL CONSIDERATIONS AND AN EXAMPLE

While there exist good methodological reasons for using any of the three measures of dimensionality already mentioned, the real test concerns their actual performance. Again, experience in using these measures has indicated that some perform better than others. The determinant measure (based on the distance matrix) gave the most consistent and reasonable results.

The possible disadvantage of this determinant measure is that it may become numerically unstable for larger matrices. For this reason, the entropy measure may be favoured.

An example of this third approach using a six objective problem cited by Greis, Wood and Steuer (1983) is given below. This is a MOLP problem and deals with the allocation of water resources to a number of different uses. Specifically, these uses are defined by the objectives, which are:

1. MUNIC - Water for municipal use
2. INDUS - Water for industrial use
3. ENERG - Water for power generation
4. RECRE - Water for recreation, i.e., reservoir level
5. EXPOR - Water for export
6. LFLOW - Control of low flow levels

Each objective is to be maximized.

Initially 15 efficient solutions were randomly generated as being representative of the efficient set. The correlation and distance matrices derived from this data source are shown in Table 7.1 on the following page. The distance matrix was calculated using the metric of [2].

Assume that a subset of four objectives is required. Since there are only 15 possible combinations of four objectives from six, a measure of dimensionality was calculated for each combination. The actual measure of dimensionality used was the determinant measure based on the distance matrix. The five solutions which scored highest on this measure were analysed to give the frequency of occurrence for each objective. Table 7.2, one page following, contains these results.

Correlation Matrix

	MUNIC	INDUS	ENERG	RECRE	EXPOR	LFLOW
MUNIC	1.000	0.402	0.160	-0.152	0.514	-0.867
INDUS	0.402	1.000	0.006	-0.333	0.601	-0.761
ENERG	0.160	0.006	1.000	-0.440	0.256	-0.102
RECRE	-0.152	-0.333	-0.440	1.000	-0.734	0.202
EXPOR	0.514	0.601	0.256	-0.734	1.000	-0.695
LFLOW	-0.867	-0.761	-0.102	0.202	-0.695	1.000

Distance Matrix

	MUNIC	INDUS	ENERG	RECRE	EXPOR	LFLOW
MUNIC	1.000	0.578	0.484	0.441	0.574	0.205
INDUS	0.578	1.000	0.454	0.386	0.671	0.271
ENERG	0.484	0.454	1.000	0.500	0.511	0.473
RECRE	0.441	0.386	0.500	1.000	0.412	0.577
EXPOR	0.574	0.671	0.511	0.412	1.000	0.346
LFLOW	0.205	0.271	0.473	0.577	0.346	1.000

Table 7.1 Correlations of Objectives

	MUNIC	INDUS	ENERG	RECRE	EXPOR	LFLOW
Frequency	4	4	4	3	1	4

Table 7.2 Frequency Scores based on 15 sets of objectives

The result of Table 7.2 suggests that a suitable combination of four objectives would be MUNIC, INDUS, ENERG and LFLOW.

Using this third approach it is possible to examine subsets of different size. However, with a little experimentation, it is expected that it will become clear which objectives significantly contribute to explaining the variability or variety, depending on the measure used. The approach could be further modified to allow the DM to specify certain objectives which must always be included in the reduced set.

7.5 CONCLUSIONS

The issue of reducing the number of objectives in a MODM has been examined, with an emphasis on analytical approaches for achieving this. It has been assumed that the MODM has already been formulated. Consequently, the analysis is performed a posteriori, i.e., after formulation, using a sample from the set of efficient solutions as a data source. This is in contrast to the more common situation where it is sought to keep the number of objectives at a minimum either before or during formulation of the MODM.

Three possible analytical approaches have been suggested, with an example given of the third approach. Three different measures of dimensionality have also been discussed, although some difficulties remain as to what constitutes a suitable analytic measure to use in reducing dimensionality. The approaches and measures discussed can best be viewed as support for the DM as he or she seeks to reduce the number of objectives and, as such, need to be kept in balance with subjective approaches which make use of preference information provided by the DM.

CHAPTER 8 CONCLUSION

In Chapter 7, a distinction was made between analytic and subjective approaches for reducing the number of objectives in a MODM that has already been formulated. This distinction between the analytic and the subjective provides a useful framework for considering the MODM solution methods.

By definition, finding the most preferred solution to a MODM requires subjective preference information from the DM. However, there is also an analytic part to any MODM solution method. Firstly, this involves distinguishing between obviously bad and obviously good solutions, e.g., efficient and inefficient solutions. And secondly, the solution method should also provide some structure within which the DM will be able to progress toward a preferred solution.

It is in this area of determining a suitable structure that both subjective and analytic considerations are relevant. As the review of behavioural issues of decision making (Chapter 3) has indicated, it is important to understand the actual decision making strategies of a DM. There must of necessity be a balance between the structure of the solution method conforming to the DM's decision strategy and the converse, where the DM's strategy is required to conform to the structure of the method. And since it should be the intention of the solution method to serve the DM, there should be greater emphasis placed on the former, i.e., where the structure of the solution method conforms to the decision strategy of the DM.

Different DM's will inevitably have different decision strategies. This has been clearly demonstrated in the experiment of Chapter 5. Although the STE method was distinctly favoured overall, some subjects did not like the method and evidenced a definite preference for other solution methods. This would suggest that there is not a single "ideal" solution method; instead the "ideal" solution method should be such that it can accommodate the different decision strategies of different DM's.

In the literature on solution methods, there has been an emphasis on those methods which interact with the DM and thereby progress toward a preferred solution. This process of interaction would seem to be an indispensable part of any practical solution method. (Even goal programming, which is not an interactive method per se, is often practically used in an interactive fashion. As the DM observes the solution which results from a given set of goals and weights, he or she then chooses another set of goals and weights and thereby explores possible solutions, i.e., interacts.) The interaction process gives the DM the opportunity to learn his or her preferences and to become familiar with the range of possible outcomes. Given that interaction with the DM is a vital ingredient in any solution method, there is then a need to find the most appropriate ways to structure this process of interaction.

As mentioned in Section 6.1.1, all solution methods are effectively seeking a match between the respective geometries of the DM's preferences and the efficient set. Some solution methods require that the DM provide information about the geometry of his or her preferences which the method then analyzes. Alternatively, the method may provide information (e.g., tradeoffs) which the DM analyzes. It is likely that the latter approach will be less restrictive in that it will give the DM more freedom to pursue his or her decision strategy. However, it was also clear from Chapter 3 that there is a relatively low ceiling on the information processing capabilities of a human DM. Although it would seem desirable to follow the second approach and allow the DM to analyze information presented by the solution method, the amount of information that is presented should be kept at a minimum. This can be achieved by having the DM provide some preference information. Many solution methods fall into this category; where the method does most of the work in providing information to the DM, with the DM required to make appropriate responses.

This thesis has focussed on a particular formulation of the MODM which has been called the P1 or maxmin formulation. This formulation is virtually the same as the Tchebycheff or P2 formulation except that it is achievement oriented rather than deviation oriented. Consequently, the results derived for the P1 formulation apply equally well to the P2 formulation.

The P1 formulation constitutes a sound analytical base for a MODM solution method. Under this formulation, a wide range of solution strategies are available. These include the tradeoff (TO) and SJT methods of Chapter 6, the Naive method as described in Chapters 4 and 5 and Steuer's (STE) method. The surrogate worth tradeoff (SWT) method can effectively be performed using the P1 formulation, since all relevant pairwise tradeoff information is available. Also, the STEM method can be used whereby objectives are constrained at each iteration. And finally, a variation of the Geoffrion, Dyer and Feinberg (GDF) method can also be incorporated using the approach of Sakawa and Mori (1983).

With the exception of the STEM method, all these other methods which can be executed under the P1 formulation are applicable to non-linear as well as linear MODM's.

DM's have been found to use both compensatory and non-conmpensatory strategies in the process of coming to a decision. A further distinction between decision strategies is that some strategies are global whereas others tend to be more incremental in nature. The following hybrid solution method, based on the P1 formulation, should be able to accommodate the different decision strategies mentioned above. This hybrid solution method (or package) would effectively contain the TO, STE and Naive solution methods as well as provision for placing upper and lower bounds on objectives at any iteration (as in the STEM method).

Using such a hybrid solution method will obviously have a higher "set up cost" than in the case of a single solution method, because there is simply more for the DM to become familiar with. However, once the DM is familiar with the features of this hybrid solution method, he or she will be in an excellent position to choose that combination of individual solution methods which are most suitable to the decision making environment and to his or her actual decision strategies. The emphasis of this hybrid method is on flexibility and aiding the DM in his or her search for a preferred solution, rather than on any guarantee of convergence to a most preferred solution.

In summary, then, this thesis has sought to consider the actual decision making behaviour of the DM and on the basis of this and the properties of the P1 formulation, to develop appropriate solution methods MODM's.

BIBLIOGRAPHY

ANDERSON, N.H. (1971) Integration theory and attitude change. Psychological Review, 78: 171-206.

ARROW, K.J. (1951) Social choice and individual values. New York, John Wiley.

AUSTIN, M.P. and K.D. Cocks (1978) Land use on the south coast of NSW. A study in methods of acquiring and using information to analyze regional land use options. CSIRO, Australia. General Report, Vol 1.

BANKER, R.L. and S.K. Gupta (1980) A process for hierarchical decision making with multiple objectives. Omega, 8: 137-149.

BAUM, S. and R.C. Carlson (1974) Multigoal optimization in management science. Omega, 2: 607-623.

BELENSON, S.M. and K.C. Kapur (1973) An algorithm for solving multicriterion linear programming problems with examples. Operational Research Quarterly, 24:65-77.

BELL, D.E. (1982) The manager's dilemma: good decisions need not lead to good outcomes. In M. Grauer, A. Lewandowski, A.P. Wierzbicki (eds.), Multiobjective and stochastic optimization. Laxenburg, Austria, IIASA Collaborative Proceedings Series. CP-82-512, pp. 99-106.

BELL, D.E., R.L. Keeney and H. Raiffa (eds.), (1977) Conflicting objectives in decisions. New York, John Wiley.

BENAYOUN, R., J. De Montgolfier, J. Tergny and O. Larichev (1971) Linear programming with multiple objective functions: step method (STEM). Mathematical Programming, 3: 366-375.

BODILY, S.E. (1979) A delegation process for combining individual utility functions. Management Science, 25: 1035-1041.

BOWMAN, V.J. (1976) On the relationship of the Tchebycheff norm and the efficient frontier of multiple-criteria objectives. In H. Thiriez and S. Zionts (eds.), Multiple criteria decision making. Jouy-en-Josas, 1975. Lecture Notes in Economics and Mathematical Systems. New York, Springer-Verlag. Vol. 130, pp. 76-86.

BRUNER, J.S. and L. Postman (1949) Perception of Incongruity. Journal of Personality, 18: 206-223.

BRUNSWIK, E. (1956) Perception and the representative design of psychological experiments. Berkeley, University of California Press.

CHADWICK, G.F. (1971) A systems view of planning. Oxford, Pergamon Press.

CHANKONG, V. and Y.Y. Haimes (1978) The interactive surrogate worth trade-off (ISWT) method for multiobjective decision making. In S. Zionts (ed.) Multiple criteria problem solving. Proceedings, Buffalo, New York (USA), 1977. Lecture Notes in Economics and Mathematical Systems. New York, Springer-Verlag. Vol. 155, pp. 42-67.

CHANKONG, V. and Y.Y. Haimes (1983a) Optimization-based methods for multiobjective decision-making: an overview. Large Scale Systems, 5: 1-33.

CHANKONG, V. and Y.Y. Haimes (1983b) Multiobjective decision making: theory and methodology. New York, Elsevier - North Holland.

CHARNES, A. and W.W. Cooper (1961) Management models and industrial applications of linear programming, Volume 1. New York, John Wiley.

CHOO, E.U. and D.R. Atkins (1980) An interactive algorithm for multicriteria programming. Computers and Operations Research, 7: 81-87.

CHURCHMAN, C.W. (1961) Prediction and optimal decision: philosophical issues of a science of values. Englewood Cliffs, N.J., Prentice Hall.

CHURCHMAN, C.W., R.L. Ackoff and E.L. Arnoff (1957) Introduction to operations research. New York, John Wiley.

COX, D.R. (1958) Planning of experiments. New York, John Wiley.

DAELLENBACH, H.G. and J.A. George (1978) Introduction to operational research techniques. Boston, Allyn and Bacon.

DALKEY, N.C. (1976) Group decision analysis. In M. Zeleny (ed.) Multiple criteria decision making. Kyoto, 1975. Lecture Notes in Economics and Mathematical Systems. New York, Springer-Verlag. Vol. 123, pp.45-74.

DALKEY, N.C. and O. Helmer (1963) An experimental application of the Delphi method to the use of experts. Management Science, 9: 456-467.

DAWES, R.M. and B. Corrigan (1974) Linear models in decision making. Psychological Bulletin, 81: 95-106.

DE KLUYVER, C.A. and F.K. Martinson (1979) Issues in formulating multiple objective linear programming problems. Proceedings of the 15th annual conference, New Zealand Operational Research Society. pp. 55-68.

DE NEUFVILLE, R. and R.L. Keeney (1972) Use of decision analysis in airport development for Mexico City. In A.W. Drake, R.L. Keeney, P.M. Morse (eds.), Analysis of public systems, Cambridge, Mass., MIT. pp. 497-519.

DE NEUFVILLE, R. and M. McCord (1983) Measurement of utility with validity: significant problems for decision analysis. Paper presented at ORSA/TIMS Conference, Orlando, Florida, U.S.A., November, 1983.

DE SAMBLANCKX, S., P. Depraetere and H. Muller (1982) Critical considerations concerning the multicriteria analysis by the method of Zionts and Wallenius. European Journal of Operational Research, 10: 70-76.

DRAPER, N.R. and H. Smith (1966) Applied regression analysis. New York, John Wiley.

DUDYCHA, L.W. and J.C. Naylor (1966) Characteristics of the human inference process in complex choice behaviour situations. Organizational Behaviour and Human Performance, 1: 110-128.

DYER, J.S. (1972) Interactive goal programming. Management Science, 19: 62-70.

ECKER, J.G. and I.A. Kouada (1978) Finding all efficient extreme points for linear multiple objective programs. Mathematical Programming, 14: 249-261.

ECKER, J.G. and N.E. Shoemaker (1981) Selecting subsets from the set of efficient vectors. SIAM Journal of Control and Optimization, 19: 505-515.

ECKER, J.G., N.S. Hegner and I.A. Kouada (1980) Generating all maximal efficient faces for multiple objective linear programs. Journal of Optimization Theory and Applications, 30: 353-381.

EDWARDS, W. (1977) How to use multiattribute utility measurement for social decision making. IEEE Transactions SMC, 17: 326-331.

EINHORN, H.J. and R.M. Hogarth (1981) Behavioural decision theory: processes of judgment and choice. Annual Review of Psychology, 32: 53-88.

EVANS, J.P. and R.E. Steuer (1973) Generating efficient extreme points in linear multiple objective programming: two algorithms and computing experience. In J.L. Cochrane and M. Zeleny (eds.), Multiple criteria decision making. Columbia, University of South Carolina Press.

FICHEFET, J. (1976) GPSTEM: An interactive multiobjective optimization method. In A. Prekopa (ed.), Progress in operations research. Amsterdam, North-Holland. Vol 1, pp.317-332.

FISCHOFF, B., P. Slovic and S. Lichenstein (1980) Knowing what you want: measuring labile values. In T.S. Wallsten (ed.), Cognitive processes in choice and decision behaviour. Hillsdale, New Jersey, Lawrence Erlbaum Associates. pp. 117-141.

FRENCH, S. (1984) Interactive multi-objective programming: its aims, applications and demands. Journal of the Operational Research Society, 35: 827-834.

GEOFFRION, A.M. (1968) Proper efficiency and the theory of vector maximization. Journal of Mathematical Analysis and Application, 22: 618-630.

GEOFFRION, A.M., J.S. Dyer and A. Feinberg (1972) An interactive approach for multi-criterion optimization. Management Science, 19: 357-368.

GIMPL, M.L. (1985) Intolerance of ambiguity and the need for forecasts. Paper presented at the 5th International Symposium on Forecasting, Montreal, Canada, 1985. Department of Business Administation, University of Canterbury, Christchurch, New Zealand.

GOICOECHEA, A., L. Duckstein and M. Fogel (1979) Multiple objectives under uncertainty: an illustrative application of PROTRADE. Water Resources Research, 15: 203-209.

GOICOECHEA, A., D.R. Hansen and L. Duckstein (1982) Multiobjective decision analysis with engineering and business applications. New York, John Wiley.

GREIS, N.P., E.F. Wood and R.E. Steuer (1983) Multicriteria analysis of water allocation in a river basin: the Tchebycheff approach. Water Resources Research, 19: 865-875.

GUPTA, R. and S.R. Arora (1977) Programming problem with maximin objective function. Opsearch, 14: 125-130.

HAIMES, Y.Y. and V. Chankong (1979) Kuhn-Tucker multipliers as trade-offs in multiobjective decision-making analysis. Automatica, 15: 59-72.

HAIMES, Y.Y. and W.A. Hall (1974) Multiobjectives in water resources systems analysis: the surrogate worth tradeoff method. Water Resources Research, 10: 615-624.

HAIMES, Y.Y., W.A. Hall and H.T. Freedman (1975) Multiobjective optimization in water resources systems: the surrogate worth trade-off method. Amsterdam, Elsevier.

HAMMOND, K.R. (1976) Externalizing the parameters of quasi-rational thought. In M. Zeleny (ed.), Multiple criteria decision making. Kyoto, 1975. Lecture Notes in Economics and Mathematical Systems. New York, Springer-Verlag. Vol. 130, pp. 75-96.

HAMMOND, K.R. and B. Brehmer (1973) Quasi-rationality and distrust: implications for international conflict. In L. Rappoport and D.A. Summers (eds.), Human judgement and social interaction. New York, Holt Rhinehart and Winston. pp. 337-391.

HAMMOND, K.R., R.L. Cook and L. Adelman (1977) Policy: an aid for decision making and international communication. Columbia Journal of World Business, 12: 79-93.

HAMMOND, K.R., T.R. Stewart, B. Brehmer and D.O. Steinmann (1975) Social judgement theory. In M.F. Kaplan, S. Schwartz (eds.), Human judgement and decision processes. New York, Academic Press. pp. 271-312.

HO, J.K. (1981) Holistic preference evaluation in multiple criteria optimization. Working paper, Management Science Program, University of Tennessee, Knoxville, Tennessee, USA.

HO, J.K. (1985) A decision support environment for totally subjective multiple criteria optimization. Working Paper, Management Science Program, University of Tennessee, Knoxville, Tennessee, USA.

HOGARTH, R.M. (1980) Judgement and choice: the psychology of decision. New York, John Wiley.

HULL, C.H. and N.H. Nie (eds.) (1981) SPSS Update 7-9: New procedures and facilities for releases 7-9. New York, McGraw-Hill.

HURSCH, C.J., K.R. Hammond and J.L. Hursch (1964) Some methodological considerations in multiple-cue probability studies. Psychological Review, 71: 42-60.

HWANG, C-L. and K. Yoon (1981) Multiple attribute decision making: methods and applications: a state of the art survey. Lecture Notes in Economics and Mathematical Systems. New York, Springer-Verlag. Vol. 186.

HWANG, C-L. and A.S. Masud (1979) Multiple objective decision making - methods and applications: a state of the art survey. Lecture Notes in Economics and Mathematical Systems. New York, Springer-Verlag. Vol. 164.

ISERMANN, H. (1977) The enumeration of the set of all efficient solutions for a linear multiple objective program. Operational Research Quarterly, 28: 711-725.

JACOBY, J., D.E. Speller and C.A. Kohn (1974) Brand choice behaviour as a function of information load. Journal of Marketing Research, 11: 63-69.

JAMESON, D. (1984) Personal communication. To appear in Central North Island Planning Area Management Plan, Department of Lands and Survey, Wellington, New Zealand.

JOHNSEN, E. (1968) Studies in multiobjective decision models. Lund, Studentlitteratur.

KAHNEMAN, D. and A. Tversky (1979) Prospect theory. An analysis of decision under risk. Econometrica, 47: 263-291.

KAPLAN, S. (1974) Application of programs with maximin objective functions to problems of optimal resource allocation. Operations Research, 22: 802-807.

KEEN, P.G.W. (1977) The evolving concept of optimality. In M.K. Starr and M. Zeleny (eds.), Multiple criteria decision making. TIMS studies in the management sciences. Vol 6. Amsterdam, North Holland. pp. 31-58.

KEENEY, R.L. and H. Raiffa (1976) Decisions with multiple objectives: preference and value tradeoffs. New York, John Wiley.

KRAFT, C.H. and C. van Eeden (1968) A nonparametric introduction to statistics. New York, Macmillan.

LEVINE, P. and J-Ch. Pomerol (1984) PRIAM, an interactive program for choosing among multiple attribute alternatives. Working Paper, Laboratoire d'econometrie de l'universite Pierre et Marie Curie, Paris, France.

LEWANDOWSKI, A. and M. Grauer (1982) The reference point optimization approach - methods of efficient implementation. In M. Grauer, A. Lewandowski and A.P. Wierzbicki (eds.), Multiobjective and stochastic optimization. Laxenburg, Austria. IIASA Collaborative Proceedings Series. CP-82-512, pp. 353-376.

LIGHTNER, M.R. and S.W. Director (1981) Multiple criterion optimization for the design of electronic circuits. IEEE Transactions CAS, 28: 169-179.

LIN, W.T. (1980) A survey of goal programming applications. Omega, 8: 115-117.

LINDBLOM, C.E. (1959) The science of muddling through. Public Administration Review, 19: 79-88.

LORD, F.M. (1962) Cutting scores and errors of measurement. Psychometrika, 27: 19-30.

MARCH, J.G. (1978) Bounded rationality, ambiguity and the engineering of choice. Bell Journal of Economics, 9: 587-608.

MARCH, J.G. and H.A. Simon (1958) Organisations. New York, John Wiley.

MARTINSON, F.K. (1977) An application of multiple objective linear programming to the formulation of management plans for multiple-use public lands. PhD Thesis, College of Business and Administration, University of Colorado, Colorado, USA.

MASUD, A.S. and C-L. Hwang (1981) Interactive sequential goal programming. Journal of the Operational Research Society, 32: 391-400.

MCGINNIS, M.A. (1984) The key to strategic planning: integrating analysis and intuition. Sloan Management Review, 25(No. 1): 45-52.

MILLER, D.W. and M.K. Starr (1967) The structure of human decisions. Englewood Cliffs, N.J., Prentice Hall.

MILLER, G.A. (1956) The magical number seven plus or minus two: some limits on our capacity to process information. Psychological Review, 63: 81-97.

MONARCHI, D.E., C.C. Kisiel and L. Duckstein (1973) Interactive multiple objective programming in water resources. Water Resources Research, 9: 11-20.

MORGENSTERN, O. (1979) Some reflections on utility. In M. Allais and O. Hagen (eds.), Expected utility hypotheses and the Allais paradox. Dordrecht, Holland, D. Reidel. pp. 175-183.

MORSE, J.N. (1980) Reducing the size of the non-dominated set: pruning by clustering. Computers and Operations Research, 7: 55-66.

MUMPOWER, J.L., V. Viers and K.R. Hammond (1979) Scientific information, social values and policy formation: the application of simulation models and judgement analysis to the Denver regional air pollution problem. IEEE Transactions SMC 9: 464-476.

NIE, N.H., C.H. Hull, J.G. Jenkins, K. Steinbrenner and D.H. Bent (1975) SPSS: Statistical package for the social sciences. New York, McGraw-Hill.

NIJKAMP, P. and P. Rietveld (1976) Multi objective programming models: new ways in regional decision making. Regional Science and Urban Economics, 6: 255-274.

NIJKAMP, P. and P. Rietveld (1981) Hierarchical multiple objective models in a spatial system. In P. Nijkamp and J. Spronk (eds.), Multiple criteria analysis - operational methods. England, Gower. pp. 163-186.

OLENIK, S.C. and Y.Y. Haimes (1979) A hierarchical multiobjective method for water resources planning. IEEE Transactions SMC, 9: 534-544.

OPPENHEIMER, K.R. (1978) A proxy approach to multi-attribute decision making. Management Science, 24: 675-689.

PAYNE, J.W. (1976) Task complexity and contingent processing in decision making: an information search and protocol analysis. Organizational Behaviour and Human Performance, 16: 366-387.

PHELPS, R.H. and J. Shanteau (1978) Livestock judges: how much information can an expert use? Organizational Behaviour and Human Performance, 21: 209-219.

POSNER, M.E. and C-T. Wu (1981) Linear max-min programming. Mathematical Programming, 20: 166-172.

READ, E.G. (1985) Personal communication.

RIETVELD, P. (1980) Multiple objective decision methods and regional planning. Amsterdam, North Holland.

ROSENTHAL, R.E. (1982) Principles of multiobjective optimization. Working paper No. 154, College of Business Administration, University of Tennessee, Knoxville, Tennessee, USA.

ROSENTHAL, R.E. (1983) Goal programming - a critique. New Zealand Operational Research, 11: 1-8.

SAATY, T.L. (1977) A scaling method for priorities in hierarchical structures. Journal of Mathematical Psychology, 15: 234-281.

SAKAWA, M. (1982) Interactive multiobjective decision making by the sequential proxy approximation technique: SPOT. European Journal of Operational Research, 9: 386-396.

SAKAWA, M. and F. Seo (1983) Interactive multiobjective decision-making in environmental systems using the fuzzy sequential proxy approximation technique. Large Scale Systems, 4: 223-243.

SAKAWA, M. and N. Mori (1983) Interactive multiobjective decision-making for non-convex problems based on the weighted Tchebycheff norm. Large Scale Systems, 5: 69-82.

SARTORIS, W.L. and M.L. Spruill (1974) Goal programming and working capital management. Financial Management, 3: 67-74.

SCHOEMAKER, P.J.H. and C.C. Waid (1982) An experimental comparision of different approaches to determining weights in additive utility models. Management Science, 28: 182-196.

SIMON, H.A. (1955) A behavioural model of rational choice. Quarterly Journal of Economics, 69: 99-118.

SIMON, H.A. (1957) Administrative Behaviour. 2d ed. New York, Macmillan.

SIMON, H.A. (1978) Rationality as process and as product of thought. American Economic Review, 68: 1-16.

SOLAND, R.M. (1979) Multicriteria optimization: a general characterization of efficient solutions. Decision Sciences, 10: 26-38.

STARR, M.K. and L.H. Greenwood (1977) Normative generation of alternatives with multiple criteria evaluation. In M.K. Starr and M. Zeleny (eds.), TIMS studies in the management sciences. Vol 6. Amsterdam, North-Holland, pp. 111-127.

STARR, M.K. and M. Zeleny (1977) MCDM - State and future of the arts. In M.K. Starr and M. Zeleny (eds.), TIMS studies in the management sciences. Vol 6. Amsterdam, North-Holland, pp. 5-29.

STEUER, R.E. (1976) Multiple objective linear programming with interval criterion weights. Management Science, 23: 305-316.

STEUER, R.E. (1977) An interactive multiple objective linear programming procedure. In M.K. Starr and M. Zeleny (eds.), TIMS studies in the management sciences. Vol 6. Amsterdam, North Holland. pp. 225-239.

STEUER, R.E. (1983) Operating manual for the ADBASE multiple objective linear programming computer package, release 2/83. Working paper series 83-130, College of Business Administration, The University of Georgia, Athens, Georgia, 30602, USA.

STEUER, R.E. (1984) Operating considerations pertaining to the interactive weighted Tchebycheff procedure. In M. Grauer and A.P. Wierzbicki (eds.), Interactive decision analysis, proceedings Laxenburg, Austria, 1983. Lecture Notes in Economics and Mathematical Systems. Berlin, Springer-Verlag. Vol. 229, pp. 132-138.

STEUER, R.E. and E.U. Choo (1983) An interactive weighted Tchebycheff procedure for multiple objective programming. Mathematical Programming, 26: 326-344.

STEUER, R.E. and F.W. Harris (1980) Intra-set point generation and filtering in decision and criterion space. Computers and Operations Research, 7: 41-53.

STEUER, R.E. and A.T. Schuler (1978) An interactive multiple objective linear programming approach to a problem in forest management. Operations Research, 26: 254-269.

STEWART, T.J. (1984) Inferring preferences in multiple criteria decision analysis using a logistic regression model. Management Science, 30: 1067-1077.

TELL, B. (1978) An approach to solving multi-person, multiple criteria decision-making models. In S. Zionts (ed.), Multiple criteria problem solving: proceedings. Buffalo, NY (USA), 1977. Lecture Notes in Economics and Mathematical Systems. New York, Springer-Verlag. Vol. 155, pp. 482-493.

TORN, A.A. (1980) A sampling-search-clustering approach for exploring the feasible/efficient solutions of multiple criteria decision making problems. Computers and Operations Research, 7: 67-79.

TUCKER, L.R. (1964) A suggested alternative formulation in the developments by Hursch, Hammond and Hursch and by Hammond, Hursch and Todd. Psychological Review, 71: 528-530.

TVERSKY, A. (1972) Elimination by aspects: a theory of choice. Psychological Review, 79: 281-299.

TVERSKY, A. and D. Kahneman (1974) Judgement under uncertainty: heuristics and biases. Science, 185: 1124-1131.

VON NEUMANN, J. and O. Morgenstern (1944) Theory of games and economic behaviour. N.J., Princeton University Press.

WALLENIUS, J. (1975) Comparative evaluation of some interactive approaches to multicriterion optimization. Management Science, 21: 1387-1396.

WALLSTEN, T.S. (1980) Processes and models to describe choice and inference behaviour. In T.S. Wallsten (ed.), Cognitive processes in choice and decision behaviour. Hillsdale, New Jersey, Lawrence Erlbaum Associates. pp. 215-238.

WHITE, D.J. (1982) The foundations of multi-objective interactive programming. In P. Hansen (ed.), Essays and surveys on multiple criteria decision making. Lecture Notes in Economics and Mathematical Systems. Berlin, Springer-Verlag. Vol. 209, pp. 406-415.

WIERZBICKI, A.J. (1980) The use of reference objectives in multiobjective optimization. In G. Fandel and T. Gal (eds.), Multiple criteria decision making, theory and application. Lecture Notes in Economics and Mathematical Systems. New York, Springer-Verlag. Vol. 177, pp. 468-486.

WINKELS, H-M. (1982) A flexible decision aid method for linear multicriteria systems. In M. Grauer, A. Lewandowski and A.P. Wierzbicki (eds.), Multiobjective and stochastic optimization. Laxenburg, Austria, IIASA Collaborative Procedings Series. CP-82-512, pp.377-410.

WINKELS, H-M. and M. Meika (1984) An integration of efficiency projections into the Geoffrion approach for multiobjective linear programming. European Journal of Operational Research, 16: 113-127.

YNTEMA, D.B. and W.S. Torgerson (1961) Man-computer cooperation in decisions requiring common sense. IRE Transactions on Human Factors in Electronics, HFE-2: 20-26.

YU, P.L. (1973) A class of solutions for group decision problems. Management Science, 19: 936-946.

YU, P.L. and M. Zeleny (1975) The set of all non dominated solutions in linear cases and a multicriteria simplex method. Journal of Mathematical Analysis and Applications, 49: 430-468.

ZELENY, M. (1974) Linear multiobjective programming. Lecture Notes in Economics and Mathematical Systems. New York, Springer-Verlag. Vol. 95.

ZELENY, M. (1976) The theory of the displaced ideal. In M. Zeleny (ed.), Multiple criteria decision making. Kyoto, 1975. Lecture Notes in Economics and Mathematical Systems. New York, Springer-Verlag. Vol. 123, pp. 153-206.

ZELENY, M. (1982) Multiple criteria decision making. New York, McGraw-Hill.

ZIONTS, S. (1982) Multiple criteria decision making: an overview and several approaches. Working Paper No. 454, School of Management, State University of New York at Buffalo, USA.

ZIONTS, S. and J. Wallenius (1976) An interactive programming method for solving the multiple criteria problem. Management Science, 22: 652-663.

ZIONTS, S. and J. Wallenius (1983) An interactive multiple objective linear programming method for a class of underlying nonlinear utility functions. Management Science, 29: 519-529.

APPENDIX 1

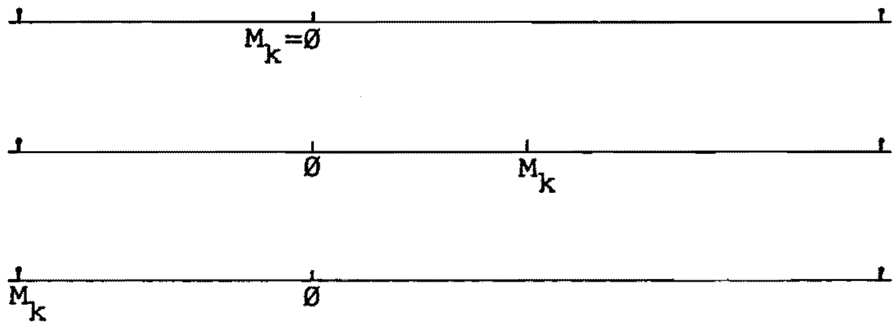
This appendix illustrates some conceptual difficulties associated with the fractional achievement norm.

These difficulties can best be illustrated by an example.

(a)

(b)

(c)



In case (a), use of the fractional achievement norm,

$$d_k^F = f_k(\underline{x}) \;/\; U_k$$

will give identical results as the range norm

$$d_k^R = (f_k(\underline{x})-M_k) \;/\; (U_k-M_k) \;.$$

In case (b), the fractional achievement norm can be misleading. If $f_k(\underline{x}) = M_k$, then the value of the norm implies an achievement of $M_k/U_k \times 100\%$. In fact, the solution is already at its worst value. Consequently, only part of the range is being used.

In case (c), if $f_k < 0$, then the resulting value of the fractional achievement norm is some negative percentage. This is a meaningless result, especially for comparative purposes.

These results can also occur when the fractional norm is based on deviations from the ideal solution, i.e.,

$$d_k = (U_k-f_k) \;/\; U_k \;=\; 1 - f_k/U_k \;.$$

APPENDIX 2

This appendix provides details of the simulation of Section 4.7.8.1. In this simulation, the P1 and P2 formulations were compared (in two dimensions) in order to examine under which formulation the closest approximation to the efficient set would be found for a tradeoff which goes beyond the current optimal basis. Figure A2.1 below reproduces Figure 4.7.

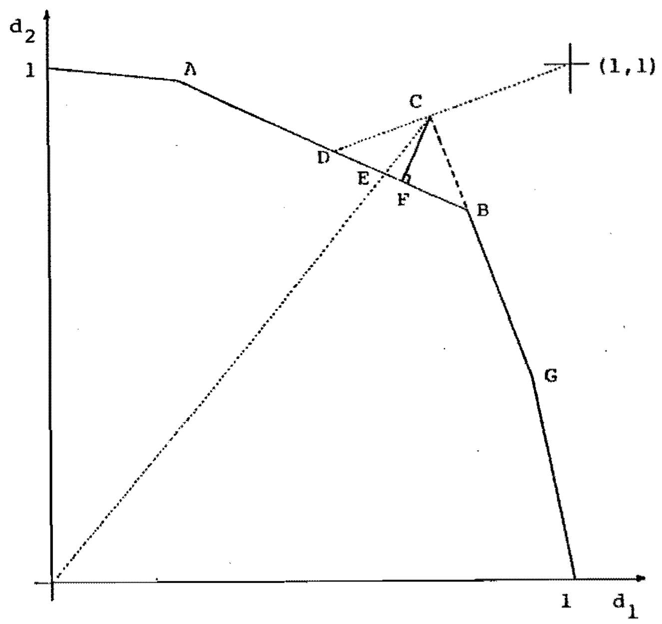


Figure A2.1

A tradeoff has occurred along BG to point C; i.e., beyond the current basis. Using solution C as weights, P1 gives solution E and P2 solution D on the line AB. Solution F is closest (in terms of Euclidean distance) to the efficient surface from C.

The simulation is performed for a number of randomly generated lines AB to ascertain which formulation was usually closer to point F. The general equation of the line AB is $\beta d_1+d_2=\alpha$. For C = (a,b), the three solutions D, E and F can be determined analytically, as shown on the following page.

$$D: \quad (d_1^D, d_2^D) = \left(\frac{1-b+(\alpha-1)(1-a)}{1-b+\beta(1-a)} \quad , \quad \frac{(b-a)+\alpha(1-b)}{1-b+\beta(1-a)} \right)$$

$$E: \quad (d_1^E, d_2^E) = \left(\frac{a}{b+\beta a} \quad , \quad \frac{b}{b+\beta a} \right)$$

$$F: \quad (d_1^F, d_2^F) = \left(\frac{\alpha\beta-\beta b+a}{\beta^2+1} \quad , \quad \frac{\alpha+\beta^2 b-\beta a}{\beta^2+1} \right)$$

The distances from the closest solution F are then given by

$$\text{dist}_k = [\ (d_1^k-d_1^F)^2 + (d_2^k-d_2^F)^2\ ]^{(1/2)}\ ,\ k \in \{D,E\}$$

The random generation of the parameters of the line AB and the point C were performed as follows.

1. Slope (β)

Any slope is deemed to be equally likely; therefore $\beta = -\tan\theta$, where θ is randomly chosen from the interval $[\emptyset, \pi/2)$.

2. Intercept (α)

In order for the randomly generated line to have contact with the unit square, $\alpha \in [1, 1+\beta]$. Consider Figure A2.2 on the following page.

The largest possible triangle AGB is generated for $\alpha = 1$. And the smallest triangle is generated when $\alpha = 1+\beta$. The line DE (which is defined by the value of α) is chosen such that

RND = (area of DGE)/(area of AGB) , RND $\in (\emptyset, 1)$

This approach means that it the average intercept value will lie closer to 1 than to $1+\beta$.

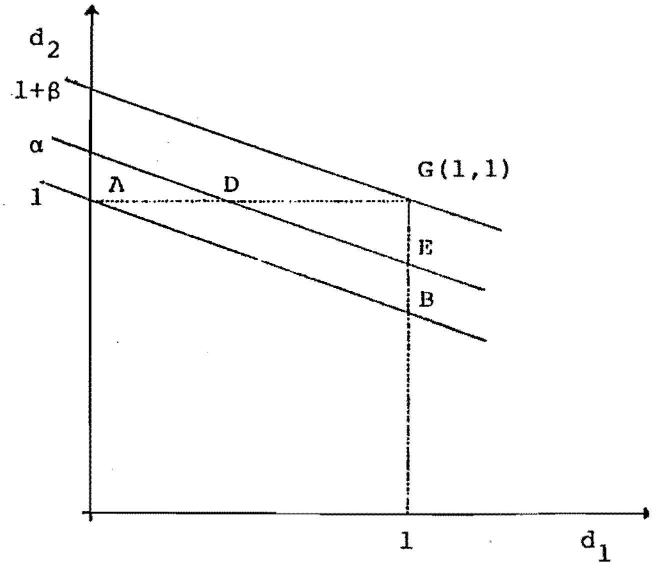


Figure A2.2

3. Point C = (a,b)

Both a and b are contained within the following intervals.

$$a \in [(\alpha-1)/\beta, 1] \quad \text{and} \quad b \in [\alpha-\beta, 1]$$

Thus a and b are randomly sampled from these intervals and rejected if $\beta a+b < \alpha$.

As stated in the body of the thesis, the result for a sample size of 3000 was that P1 was closer than P2 to F 81% of the time. The average distance that the two solutions D and E were away from the closest solution F was 0.021 for P1 and 0.090 for P2; a difference of a factor of four.

APPENDIX 3

A3.Ø INTRODUCTION

This appendix contains additional information for the experiment of Chapter 5. Specifically, the contents of this appendix are as follows:

A3.1 - Case Study: contains all details about the small manufacturing company on which the four MOLP's are based.

A3.2 - MOLP Models: contains the formulation of the MOLP model of the case study, and the model parameters for each of the four planning situations, i.e., normal, pessimistic, optimistic and conservative.

A3.3 - Explanation of each Solution Method: contains the brief explanation of each of the four solution methods which was shown to each subject before the method was actually used.

A3.4 - Demonstration of each Solution Method: contains a demonstration run of each solution method based on the normal planning situation.

A3.5 - Questionnaires: contains the two questionnaires used to obtain preference information from each subject.

A3.6 - Raw Data: contains all the raw data scores for each subject and each criterion.

A3.1 THE CASE STUDY

The following eight pages contain the case study which was given to each subject.

CASE STUDY INFORMATION

You are asked to play the role of managing director of a fictitious company "Brite-Lite Ltd" which is located on the West Coast of the North Island.

A. INFORMATION ABOUT THE COMPANY.

1 Product.

The company's factory produces various electrical components that are used in lamps, and since the production processes and the use of the components are very similar, Brite-Lite Ltd will be considered as a one product company.

2 Demand for Lamp Components.

With increasing international competition, the demand for the product is expected to decrease by about 25% over the next two years. Recent economic forecasts predict an upsurge in demand by the middle of 1986. Some reasonably reliable demand estimates have been constructed based on slightly differing assumptions concerning the development of the industry.

3 Production Capacity.

With the diminished demand for the product, Brite-Lite would normally have no constraints on production. However the company is extremely dependent on raw material suppliers and current difficulties in the procuring of these raw materials indicate that it may not be possible to satisfy the diminished demand in each month. Consequently, an inability to meet demand is expected in some months, resulting in stockouts.

4 Customers.

Brite-Lite has had a rather established clientele and contacts were made with the most important customers many years ago. Delivery times have been flexible and delays (i.e., stockouts) have not usually caused the company any significant costs, because the customers have been able to wait for their order. Due to increasing competition, customers have started demanding a better service. And it is feared that continuing delays in supply may cause some of the old customers to switch to more responsive suppliers.

5 Competition and Sales.

This branch of the industry has three competing companies, the other two larger than Brite-Lite in terms of total sales. Last year Brite-Lite's total sales were $3.1 million and, fearing a price war, the unit price of a product was fixed at $20.00. At the end of last year the directors made a decision to increase it by $1.00 to compensate for anticipated increases in wages and salaries.

6 Financial.

During the last few years, Brite-Lite has had difficulties financing it's operations, largely because one of the major finance houses thinks that the company has exceeded it's debt capacity. Consequently, negotiations for a loan of $130,000 failed last month. Contacts with other financial institutions have not, as yet, produced any satisfactory results.

The majority of the share capital (55 %) is owned by the family of the managing director who have been reluctant to issue new shares for fear of losing control of the company. But unless operating costs are significantly reduced or the company succeeds with a new loan, a new share issue will be necessary.

7 Costs.

Operating costs are estimated as follows. Labour costs, which make up 60 % of the total costs are $6.50/hr for ordinary time and $9.75/hr for overtime. Inventory holding costs are $2.90 per product per year, with a fixed holding cost of $75,000 per year.

8 Employees and the Labour Situation.

Last year there were 110 employees who worked a total of 30,000 man-days, producing about 5 units of product per man-day. The factory has been operating on a single shift basis and plans to adopt a two shift system have been abandoned because of the decrease in demand. The factory is an important employer in the town and new labour has been relatively easy to hire. Although the wages paid by the company have in general corresponded to wages paid by other companies in the area, they are below the industry average. A tentative agreement concerning wage increases has been reached with the union.

Because of the agreement with the unions, the directors do not consider it feasible to fire employees over the next year or two, even though such action is legal. Alternatively, it is possible to temporarily lay off employees if raw material shortages and decreasing demand force a cut in production. This would be achieved by a 3 or a 4 day working week. Continuing to lay off employees will have the adverse effects of increasingly negative attitudes toward the company and a probable decrease in productivity, as well as the problem of recruiting competent workers in the future.

B. A MODEL OF COMPANY OPERATIONS

A simple mathematical model of the factory operation has been constructed. It differs from a linear programming production scheduling model (minimizing costs subject to constraints on inventory and production), in that it has three conflicting objective functions instead of one. A model having multiple objectives was formulated, because it turned out to be very difficult and arbitrary to estimate in dollars both the costs due to stockouts and the loss of goodwill due to laying off employees.

The following three decision criteria were chosen as the objective functions of the model.

1st objective - Operating Costs

Minimize total operating costs, measured in dollars. As costs increase, so also does the likelihood of the managing director having to issue new shares in order to raise additional working capital to avoid possible bankruptcy. This issue of new shares is at the risk of losing control of the company.

2nd objective - Average Number of Stockouts

Minimize the average number of stockouts, calculated as a percentage of the demand out of stock [1]. As the number of stockouts increase, customers are likely to turn to more responsive suppliers. (Note: as managing director, you may be able to convince your customers to accept this situation for this year with an improvement expected next year.)

3rd objective - Number of Employees Temporarily Laid Off

Minimize the number of employees temporarily laid off, calculated as an average percentage of the total labour force laid off [2]. As the level of this objective increases, the company can expect increasing difficulty in future wage negotiations and a negative attitude from employees.

All objective function values cover the entire planning horizon of one year. Note that only the first objective is measured in monetary units.

[1] Example. If the total demand is 150,000 units and if stockouts are 3000 units in the second period, 7000 in the fifth period and 5000 in the sixth period, then the stockout percentage is 10%, i.e., (3000 + 7000 + 5000) / 150,000.

[2] Example. If 25% of the employees are laid off in the third period, 15% in the fourth period and 30% in the final period, then the average percentage of people laid off is 10%, i.e., (25 + 15 + 30) / 7 periods

Model Constraints.

Firstly there are production constraints to ensure that production does not exceed available capacity. Also, the inventory at the end of each period must be equal to the production of that period minus demand plus the inventory at the end of the previous period. Stockouts which occur in one period must be made up in subsequent periods with an upper limit on the level of stockouts for the final period. In addition, the model has constraints defining labour and employees laid off and an upper limit on operating costs. Finally labour resources used should not exceed available labour (including overtime).

The planning horizon of one year consists of seven periods of equal length.

C. THE PLANNING SITUATIONS

Based on the general information concerning the company and it's environment, four independent planning situations have been developed.

It is your task as managing director to find a compromise solution (in objective function values), i.e., a production, inventory and labour plan which is best in terms of your preferences.

The four planning situations follow.

NORMAL Planning Situation

Period	1	2	3	4	5	6	7	Total
Demand	19.0	26.3	16.5	17.5	24.4	17.6	14.0	135.3
Capacity	21.9	22.5	16.5	16.5	24.0	17.3	17.0	135.7

Wages are expected to increase from \$6.50/hr to \$8.00/hr over the year.

There is an upper limit on stockouts of 4000 units at the end of the year.

There is an upper limit on costs of \$1,787,500

From the mathematical model of Brite-Lite Ltd. under the Normal planning situation, the following information on the range of values for each objective is available.

	WORST(1)	BEST
Costs (\$)	\$1,787,500	\$1,538,069
Stockouts(%)	18.8%	6.4%
Labour Laid off (%)	12.4%	0.0%

(1) i.e., at least as bad as

Please have a good look at the range of values in order to get a good idea of what you would like for a compromise solution BEFORE you start using the solution method.

PESSIMISTIC Planning Situation

Period	1	2	3	4	5	6	7	Total
Demand	19.0	26.3	14.5	16.0	24.4	13.8	12.0	126.0
Capacity	21.9	22.5	14.5	14.0	24.0	13.0	15.0	124.9

Demand is reduced 7% from the normal situation.

In order to reduce labour costs, it has been decided to reduce the wage increases over the year. As a result, wages should increase from $6.35/hr to $7.00/hr.

Fearing a rise in the world market prices for raw materials (causing additional costs of $65,000 to $100,000) an absolute upper limit of $1,553,950 has been set for the first objective.

Consequently, the upper limit for stockouts at the end of the year has been increased to 5000 units.

From the mathematical model of Brite-Lite Ltd. under the Pessimistic planning situation, the following information on the range of values for each objective is available.

	WORST(1)	BEST
Costs ($)	$1,553,950	$1,328,090
Stockouts(%)	22.9%	10.5%
Labour Laid off (%)	19.4%	4.6%

(1) i.e., at least as bad as

Please have a good look at the range of values in order to get a good idea of what you would like for a compromise solution BEFORE you start using the solution method.

OPTIMISTIC Planning Situation

Period	1	2	3	4	5	6	7	Total
Demand	19.Ø	26.3	17.5	19.5	24.4	18.3	16.Ø	141.Ø
Capacity	22.5	23.Ø	15.5	16.5	24.Ø	17.3	18.1	136.9

Demand is estimated to be 4% larger than in the normal situation.

It is possible to extend the production capacity beyond that given in the table above at an average cost of $7.8Ø per product.

Since the company has succeeded in getting a loan of $155,ØØØ, a new upper limit of $2,Ø34,5ØØ has been set for costs.

Consequently, the upper limit for stockouts allowed at the end of the year has been lowered to 25ØØ units.

From the mathematical model of Brite-Lite Ltd. under the Optimistic planning situation, the following information on the range of values for each objective is available.

	WORST(1)	BEST
Costs ($)	$2,Ø34,5ØØ	$1,799,ØØ5
Stockouts(%)	1Ø.Ø%	Ø.Ø%
Labour Laid off (%)	8.5%	Ø.Ø%

(1) i.e., at least as bad as

Please have a good look at the range of values in order to get a good idea of what you would like for a compromise solution BEFORE you start using the solution method.

CONSERVATIVE Planning Situation

Period	1	2	3	4	5	6	7	Total
Demand	19.0	26.3	15.0	16.5	24.4	14.3	13.0	128.6
Capacity	21.9	22.5	15.5	14.5	24.0	13.3	16.0	127.7

Demand is expected to be about 5% smaller than in the normal situation.

The reduction in capacity reflects the production manager's conviction that employees are working at rate below that indicated by the estimated production capacities in the normal situation.

An upper limit on operating costs has been set at $1,697,150 and the upper limit for stockouts raised to 4350 units in the final period.

From the mathematical model of Brite-Lite Ltd. under the Conservative planning situation, the following information on the range of values for each objective is available.

	WORST(1)	BEST
Costs ($)	$1,697,150	$1,410,068
Stockouts(%)	18.3%	8.5%
Labour Laid off (%)	17.6%	0.5%

(1) i.e., at least as bad as

Please have a good look at the range of values in order to get a good idea of what you would like for a compromise solution BEFORE you start using the solution method.

A3.2 THE MOLP MODELS USED IN THE EXPERIMENT

A brief discussion of the MOLP models employed in the experiment was given in Section 5.1. These models are explained in more detail in this appendix. The four sets of parameter values needed for replicating the experiment are also provided. The sole criterion for choosing them was the "reasonableness" of the resulting objective function values. This information was generously provided by J. Wallenius, University of Jyväskylä, Finland upon request. It has been modified for the New Zealand situation.

A3.2.1 Formulation

Variables:

I_t^+ = inventory at the end of period t (product units),

I_t^- = stockouts at the end of period t (product units),

I_t = $I_t^+ - I_t^-$ (if $I_t^+ > 0$, $I_t^- = 0$ and vice versa) (product units),

INV_t = investment in period t (product units),

LF_t = size of labour force in period t (man-days),

O_t = overtime in period t (O_t^+ = expensive overtime, O_t^- = slack in constraint [A3.8]) (man-days),

OFF_t = employees laid off in period t (man-days), and

P_t = production in period t (product units).

Constants:

a^o = fixed cost of holding inventory (\$),

a_t = variable cost of holding inventory in period t (\$ per product unit),

b_t = cost of regular payroll in period t (\$ per man-day),

c_t = overtime cost in period t (assuming that overtime

$O_t \leq k_t$ (\$ per man-day),

d_t = demand in period t (d stands for the total demand) (product units),

g_t = cost of investment in period t (\$ per product unit),

h = labour input (man-days per product unit),

I_o = initial inventory (product units),

I_T = closing inventory in the final period t (product units),

k_t = amount of overtime in period t that produces a rise of Δc_t in overtime cost (man-days),

l_t = upper limit for production capacity in period t (product units),

LF^o = maximum number of regular man-days worked during the planning horizon, and

m_t = upper limit for investments in period t (product units).

Objective Functions:

Minimize total costs: [A3.1]

$$OBJ_1 = a^o + \sum_{t=1}^{T} a_t I_t^+ + \sum_{t=1}^{T} b_t LF_t + \sum_{t=1}^{T} c_t O_t + \sum_{t=1}^{T} \Delta c_t O_t^+ + \sum_{t=1}^{T} g_t INV_t$$

Minimize average stockouts: [A3.2]

$$OBJ_2 = (100/d) \sum_{t=1}^{T} I_t^-$$

Minimize employees laid off: [A3.3]

$$OBJ_3 = (100/LF^o) \sum_{t=1}^{T} OFF_t$$

Constraints:

Inventory :	$I_t = P_t - d_t + I_{t-1}$,	$t = 1,2,\ldots,T$	[A3.4]
	$I_t = I_t^+ - I_t^-$,	"	[A3.5]
Labour	$hP_t - LF_t - O_t \leq \emptyset$,	"	[A3.6]
Force :	$LF_t + OFF_t = LF^o / T$,	"	[A3.7]
Overtime :	$O_t - O_t^+ + O_t^- = k_t$,	"	[A3.8]
Prod^n.Cap. :	$P_t - INV_t \leq l_t$,	"	[A3.9]
Investments:	$INV_t \leq m_t$,	"	[A3.10]

and upper bound constraints for total costs and for stockouts at the end of the horizon, and the usual non negativity constraints

I_t^+, I_t^-, INV_t, LF_t, O_t, O_t^+, O_t^-, OFF_t, $P_t \geq \emptyset$, $t=1,2,\ldots,T$

I_t unrestricted in sign for $t = 1,2,\ldots,T$.

The models can be simplified by eliminating variables P_t, I_t and LF_t from [A3.4], [A3.5] and [A3.6] , and by dropping a number of inefficient variables. The final models to be used had 19 (20) rows and 25 (26) variables over the seven period planning horizon. The larger MOLP was for the optimistic planning situation, while the other three planning situations were incorporated in the smaller MOLP model.

A3.2.2 The Parameters of the MOLP Models

Notation:

LE = less than or equal to

GE = greater than or equal to

EQ = equal to

N = "normal" planning situation

P = "pessimistic" planning situation

C = "conservative" planning situation

Z = number of zeros.

"Normal","Pessimistic" and "Conservative" Planning Situations

Constraint matrix by rows:

1 : 1.,24Z LE 29ØØ. (N,P,C)

2 : 1.,-1.,4Z,1.,18Z GE 38ØØ. (N,P,C)

3 : 1Z,-1.,1.,3Z,1.,-1.,17Z LE Ø. (N,P), 5ØØ. (C)

4 : 2Z,1.,-1.,3Z,-1.,1.,16Z GE 1ØØØ.(N), 2ØØØ. (P,C)

5 : 3Z,1.,-1.,3Z,-1.,1.,15Z GE 4ØØ. (N,P,C)

6 : 4Z,1.,-1.,3Z,-1.,1.,14Z GE 3ØØ.(N), 8ØØ.(P), 11ØØ.(C)

7 : 5Z,-1.,4Z,1.,-1.,13Z LE 3ØØØ. (N,P,C)

8 : .2,11Z,-1.,12Z LE 485. (N,P,C)

9 : .2,-.2,4Z,.2,6Z,1.,11Z GE 975. (N,P,C)

1Ø: 1Z.-.2,.2,3Z,.2,-.2,13Z,1.,3Z LE 985.(N), 1385.(P), 1285.(C)

11: 2Z,-.2,.2,3Z,.2,-.2,13Z,1.,2Z LE 785.(N), 1Ø85.(P), 985.(C)

12: 3Z,.2,-.2,3Z,-.2,.2,4Z,1.,1ØZ GE 595. (N,P,C)

13: 4Z,-.2,.2,3Z,.2,-.2,12Z,1.,1Z LE 765.(N), 1525.(P), 14Ø5.(C)

14: 5Z,-.2,4Z,.2,-.2,12Z,1. LE 1485.(N), 1885.(P), 1685.(C)

15: 12Z,1.,2Z,-1.,1.,8Z EQ 5Ø. (N,P,C)

16: 13Z,1.,3Z,-1.,1.,6Z EQ 125. (N,P,C)

17: 14Z,1.,4Z,-1.,1.,4Z EQ 275. (N,P,C)

18: 11Z,1.,13Z LE 4000.(N), 5000.(P), 4350.(C)

19: 2.6,2.6,2.86,2.86,3.12,3.12,6Z,78.,81.25,82.5,13.,1Z,13., 1Z,13.,1Z,-45.5,-55.25,-59.8,-65. LE 46702.5(N), 78676. (P; reverse both the inequality and the sign of each coefficient), 7150.(C).

Objective functions:

OBJ_1 = 2.6,2.6,2.86,2.86,3.12,3.12,6Z,78.,81.25,82.5,13.,1Z, 13.,1Z,13.,1Z,-45.5,-54.6,-59.8,-65. constant=1,740,797.5 (N)

OBJ_1 = 2.6,2.6,2.86,2.86,3.12,3.12,6Z,78.,81.25,82.5,13.,1Z, 13.,1Z,13.,1Z,-41.6,-50.7,-53.3,-55.9. constant=1,612,676. (P)

OBJ_1 = 2.6,2.6,2.86,2.86,3.12,3.12,6Z,78.,81.25,82.5,13.,1Z, 13.,1Z,13.,1Z,-45.5,-54.6,-59.8,-65. constant=1,690,000. (C)

OBJ_2=6Z,.000739,.000739,.000739,.000739,.000739,.000739,13Z(N)

OBJ_2=6Z,.000794,.000794,.000794,.000794,.000794,.000794,13Z(P)

OBJ_2=6Z,.000750,.000750,.000750,.000750,.000750,.000750,13Z(C)

OBJ_3 = 21Z,.00333,.00333,.00333,.00333 (N,P,C)

List of decision variables:

$I_1^+, \ldots, I_6^+, I_2^-, \ldots, I_7^-, O_1, O_2, O_5, O_1^+, O_1^-, O_2^+, O_2^-, O_5^+, O_5^-, OFF_3, OFF_4, OFF_6, OFF_7$.

"Optimistic" Planning Situation

Constraint matrix by rows:

1 : 1.,25Z LE 35ØØ.

2 : 1.,-1.,4Z,1.,19Z GE 33ØØ.

3 : 1Z,1.,-1.,3Z,-1.,1.,7Z,1.,1ØZ GE 2ØØØ.

4 : 2Z,1.,-1.,3Z,-1.,1.,7Z,1.,9Z GE 3ØØØ.

5 : 3Z,1.,-1.,3Z,-1.,1.,16Z, GE 4ØØ.

6 : 4Z,1.,-1.,3Z,-1.,1.,6Z,1.,8Z GE 1ØØØ.

7 : 5Z,-1.,4Z,1.,-1.,14Z LE 21ØØ.

8 : .2,11Z,-1.,13Z LE 485.

9 : .2,-.2,4Z,.2,6Z,1.,12Z GE 975.

1Ø: 1Z,-.2,.2,3Z,2.,-.2,14Z,1.,3Z LE 785.

11: 2Z,-.2,.2,3Z,.2,-.2,14Z,1.,2Z LE 385.

12: 3Z,.2,-.2,3Z,-.2,.2,4Z,1.,11Z GE 595.

13: 4Z,-.2,.2,3Z,.2,-.2,13Z,1.,1Z LE 625.

14: 5Z,-.2,4Z,.2,-.2,13Z,1. LE 1Ø85.

15: 13Z,1.,4Z,-1.,1.,6Z EQ 15Ø.

16: 14Z,1.,5Z,-1.,1.,4Z EQ 275.

17: 15Z,1.,1ØZ LE 31ØØ.

18: 16Z,1.,9Z LE 24ØØ.

19: 11Z,1.,14Z LE 25ØØ.

2Ø: 2.6,2.6,2.86,2.86,3.12,3.12,6Z,91.,94.25,97.5,7.15,7.8,
8.45,26.,1Z,26.,-49.4,-54.6,-59.8,-65. LE 159991.

Objective Functions:

OBJ_1= 2.6,2.6,2.86,2.86,3.12,3.12,6Z,91.,94.25,97.5,7.15,7.8,
8.45,26.,1Z,26.,-49.4,-54.6,-59.8,-65. (constant = 1,874,5Ø9)

OBJ_2= 6Z,.0007009,.0007009,.0007009,.0007009,.0007009,
.0007009,14Z

OBJ_3= 22Z,.00333,.00333,.00333,.00333.

List of decision variables:

$I_1^+,\ldots,I_6^+,\ I_2^-,\ldots,I_7^-,\ O_1,\ O_2,\ O_5,\ INV_3,\ INV_4,\ INV_6,$
$O_2^+,\ O_2^-,\ O_5^+,\ O_5^-,\ OFF_3,\ OFF_4,\ OFF_6,\ OFF_7$.

The number of efficient extreme point solutions for each MOLP problem is as follows.

Normal	-	55
Pessimistic	-	35
Optimistic	-	73
Conservative	-	41

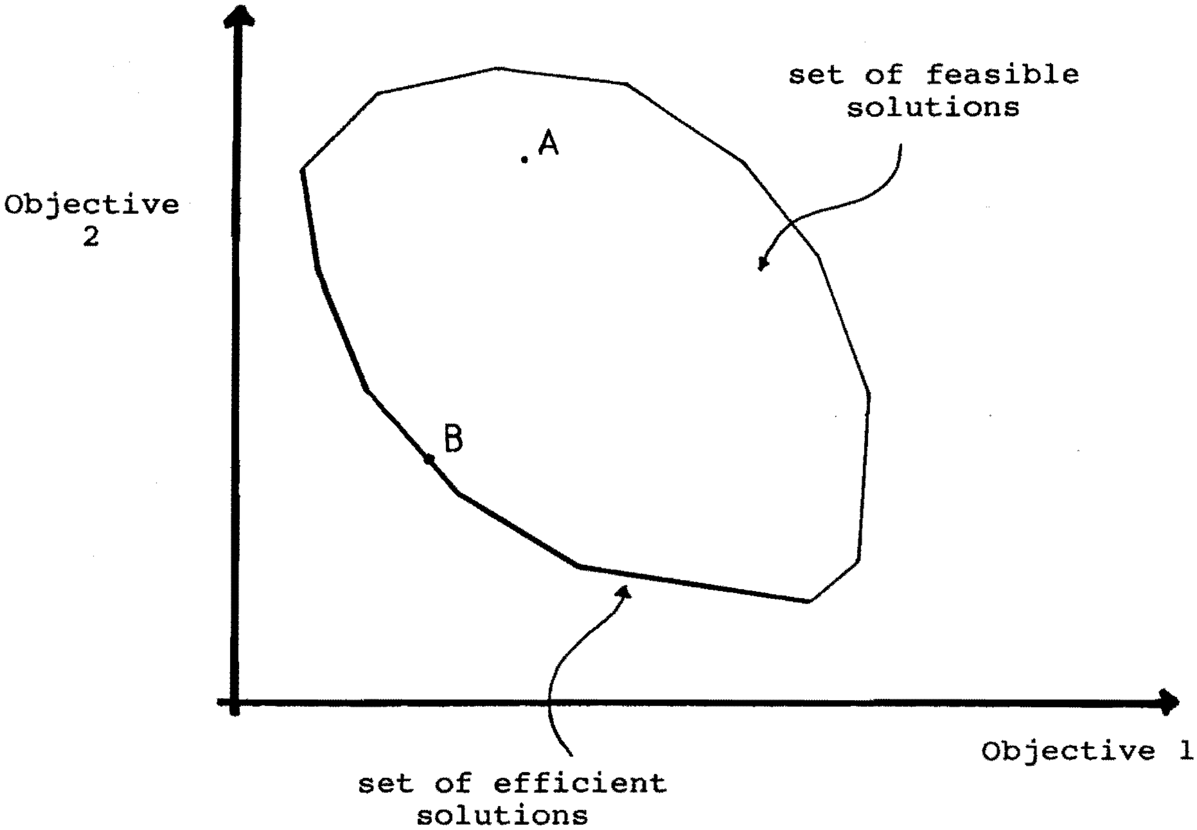
These were found using the ADBASE code, Steuer(1983).

A3.3 EXPLANATION OF EACH SOLUTION METHOD

The following five pages contain first a brief explanation of the feasible set of the MOLP model, and second, a reasonably intuitive explanation of how each of the four solution methods actually work. Each explanation was based on a simple diagram in two dimensional objective space, and presented to each subject before the solution method was actually used.

A GRAPHICAL REPRESENTATION OF THE MODEL

-with two minimizing objectives



Note: An inefficient solution is one that is feasible, but where it is possible to improve all objectives and still remain feasible. (Solution A)

An efficient solution is simply any feasible solution that is not inefficient. (Solution B)

Method MOLP/ZW

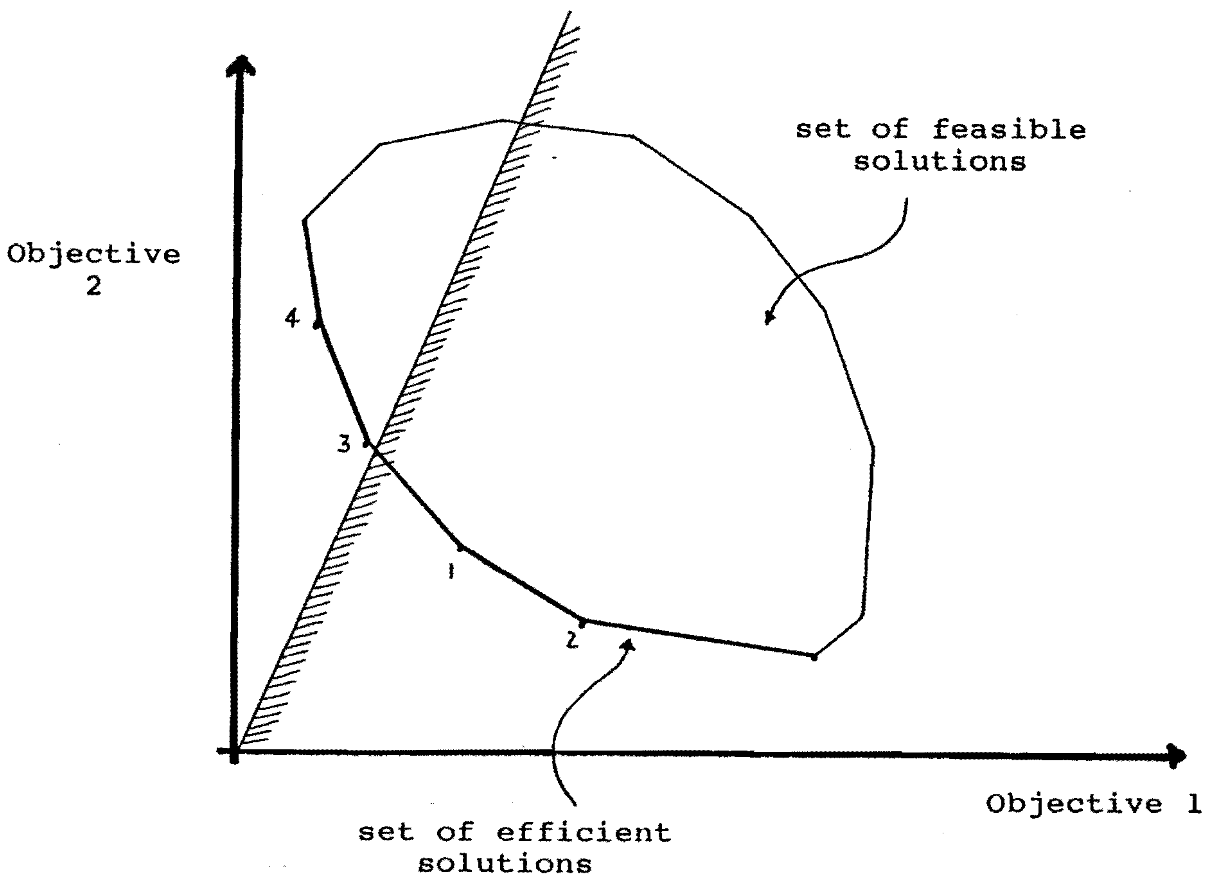
Initially a single solution is found.

Solutions which are close to this initial solution are found and the DM is required to choose between each of these close solutions and the initial solution. This information is used to exclude other solutions which will obviously not be preferred, even though the DM has not seen them.

A second solution is found consistent with the previous choices of the DM.

Close solutions are again found and the process repeats.

The process continues until there is only one solution left which is consistent with all previous choices.



Method MOLP/Naive

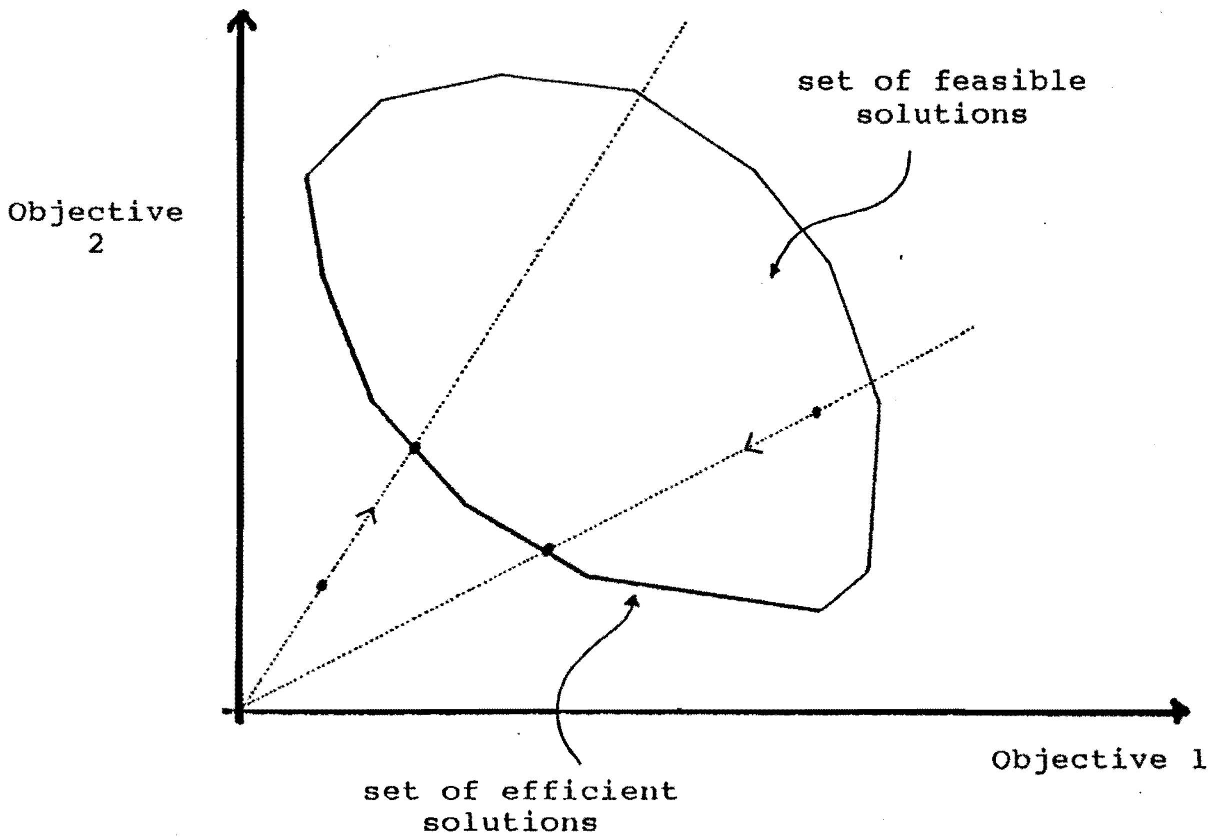
The DM chooses or guesses at a solution.

If the guess is feasible it is improved so as to be made efficient while preserving the relative proportions of each objective.

If the guess is not feasible, an efficient solution is found again preserving the relative proportions of each objective.

The DM chooses a second solution.

The process continues until the DM is satisfied with the resulting solution.



Method MOLP/SWT

This method operates in a pairwise fashion, i.e., only considering two objectives at any one time. With each pair of objectives there is an additional piece of information, which is the pairwise tradeoff.

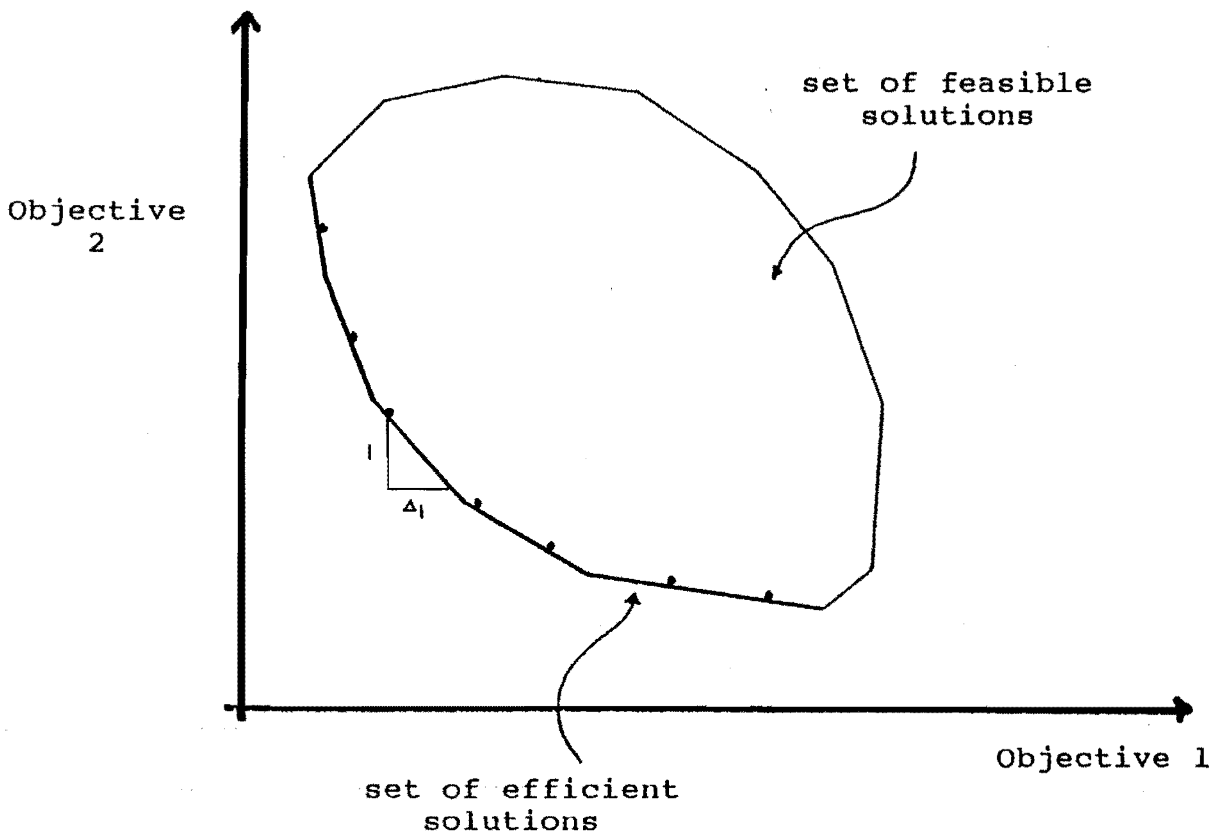
The pairwise tradeoff measures the increase in objective 1 if objective 2 is decreased by one unit.

Seven pairwise solutions and their tradeoffs are presented to the DM. The DM is required to provide a value between -10 and +10 which is his or her measure of the worth of the tradeoff.

From this information the most preferred level of objective 2 is found and objective 2 is fixed at that level.

The process repeats using pairs of objective 1 and objective 3.

Finally, the value for objective 1 is found that is consistent with the previous choices and the process stops.



Method MOLP/STE

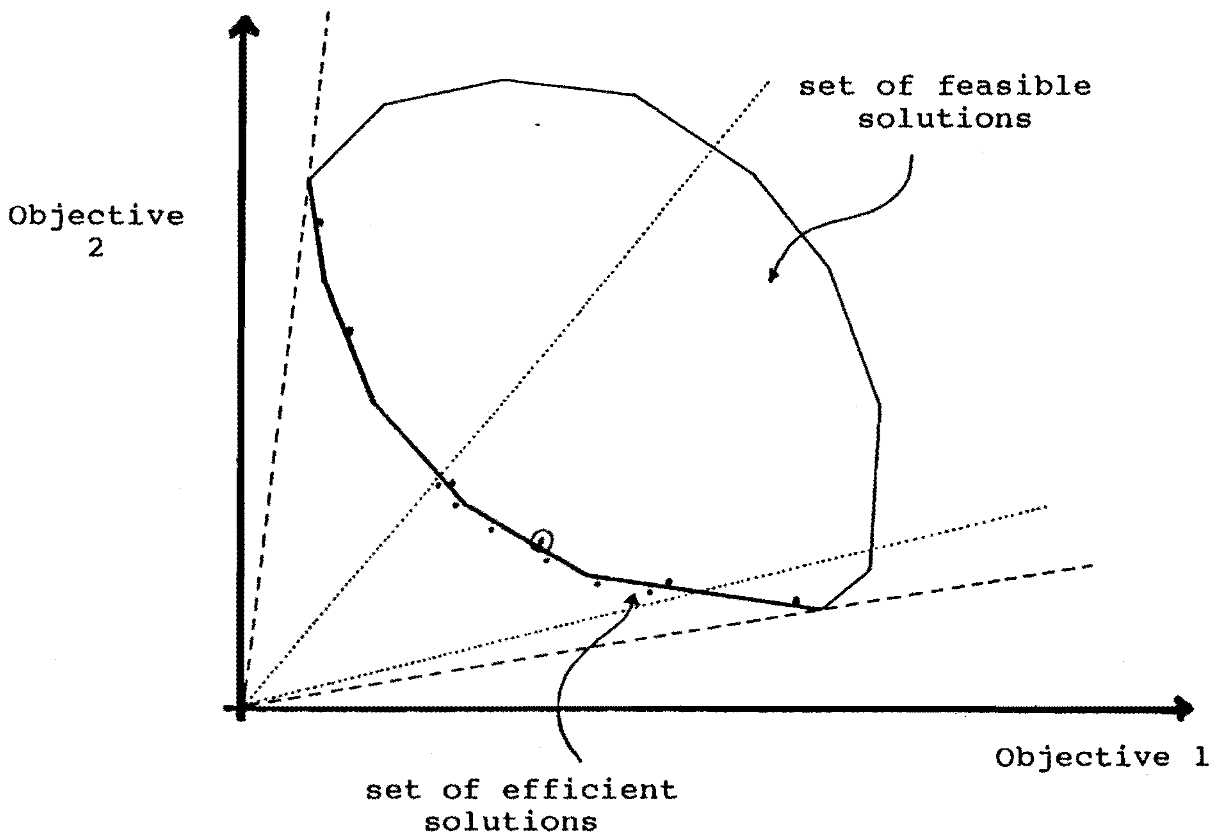
Initially six solutions are shown to the DM. These six solutions are representative of the entire set of efficient solutions.

The DM chooses one of these.

The set of efficient solutions is reduced in size by a factor of Ø.6 and a further six solutions are presented to the DM. These solutions are representative of this reduced set of efficient solutions and are centred around the solution chosen by the DM.

Again the DM chooses one of these solutions.

The process continues until the DM is satisfied with a particular solution.



A3.4 DEMONSTRATION OF EACH METHOD

This section provides details as regards the actual presentation of the solution method to each subject. Each screen is presented as it would be seen by the subject seated at the computer terminal. The rationale behind the decision making in these examples is to achieve a solution with average to below average values for costs and labour laid off. Stockouts were not considered to be as important as these other two objectives.

A3.4.1 The ZW Method

Begins on the following page.

```
RUN MOLP/ZW
#RUNNING 3132
#?
Enter name of data file (TMAT__) plus fullstop,
TMATNM/ZW.
```

```
SOLVING TO FIND SOME CLOSE SOLUTIONS
------------------------------------
- Please  wait -
```

```
  PLANNING MODEL for XYZ Co. Ltd  -  Normal Situation

                         COST          STKOUT        LABOUR
                        dollars     av % short    % laid off
                       ----------------------------------------

Current Solution      $1,551,150.     11.08%        14.39%

                       ----------------------------------------

Alternative Solution  $1,592,750.      8.72%        12.25%

Is this solution preferred to the current one?
    Enter  1  -  YES
           2  -  NO
           3  -  INDIFFERENT
1
```

PLANNING MODEL for XYZ Co. Ltd - Normal Situation

	COST dollars	STKOUT av % short	LABOUR % laid off
Current Solution	$1,551,150.	11.08%	14.39%
Alternative Solution	$1,558,950.	9.31%	15.18%

Is this solution preferred to the current one?
Enter 1 - YES
2 - NO
3 - INDIFFERENT
2

SOLVING TO FIND SOME CLOSE SOLUTIONS

- Please wait -

PLANNING MODEL for XYZ Co. Ltd - Normal Situation

	COST dollars	STKOUT av % short	LABOUR % laid off
Current Solution	$1,592,750.	8.72%	12.25%
Alternative Solution	$1,608,350.	6.95%	12.25%

Is this solution preferred to the current one?
Enter 1 - YES
2 - NO
3 - INDIFFERENT
2

PLANNING MODEL for XYZ Co. Ltd - Normal Situation

	COST dollars	STKOUT av % short	LABOUR % laid off
Current Solution	$1,592,750.	8.72%	12.25%
Alternative Solution	$1,637,568.	8.72%	8.97%

Is this solution preferred to the current one?
Enter 1 - YES
2 - NO
3 - INDIFFERENT
1

SOLVING TO FIND SOME CLOSE SOLUTIONS

- Please wait -

PLANNING MODEL for XYZ Co. Ltd - Normal Situation

	COST dollars	STKOUT av % short	LABOUR % laid off
Current Solution	$1,614,330.	11.77%	8.97%
Alternative Solution	$1,609,650.	12.77%	8.97%

Is this solution preferred to the current one?
Enter 1 - YES
2 - NO
3 - INDIFFERENT
1

PLANNING MODEL for XYZ Co. Ltd - Normal Situation

	COST dollars	STKOUT av % short	LABOUR % laid off
Current Solution	$1,614,330.	11.77%	8.97%
Alternative Solution	$1,637,568.	8.72%	8.97%

Is this solution preferred to the current one?
Enter 1 - YES
2 - NO
3 - INDIFFERENT
2

PLANNING MODEL for XYZ Co. Ltd - Normal Situation

	COST dollars	STKOUT av % short	LABOUR % laid off
Current Solution	$1,614,330.	11.77%	8.97%
Alternative Solution	$1,668,111.	11.77%	5.69%

Is this solution preferred to the current one?
Enter 1 - YES
2 - NO
3 - INDIFFERENT
1

SOLVING TO FIND SOME CLOSE SOLUTIONS

- Please wait -

PLANNING MODEL for XYZ Co. Ltd - Normal Situation

	COST dollars	STKOUT av % short	LABOUR % laid off
Current Solution	$1,640,243.	18.64%	5.69%
Alternative Solution	$1,650,399.	15.87%	5.69%

Is this solution preferred to the current one?
Enter 1 - YES
2 - NO
3 - INDIFFERENT
1

PLANNING MODEL for XYZ Co. Ltd - Normal Situation

	COST dollars	STKOUT av % short	LABOUR % laid off
Current Solution	$1,640,243.	18.64%	5.69%
Alternative Solution	$1,689,578.	18.64%	2.95%

Is this solution preferred to the current one?
Enter 1 - YES
2 - NO
3 - INDIFFERENT
2

SOLVING TO FIND SOME CLOSE SOLUTIONS

- Please wait -

```
   PLANNING MODEL for XYZ Co. Ltd  -  Normal Situation

                                COST        STKOUT       LABOUR
                               dollars    av % short   % laid off
                           ----------------------------------------

Current Solution           $1,650,399.      15.87%        5.69%

                           ----------------------------------------

Alternative Solution       $1,654,949.      14.76%        5.69%

 Is this solution preferred to the current one?
     Enter  1  -  YES
            2  -  NO
            3  -  INDIFFERENT
 2
```

```
     PLANNING MODEL for XYZ Co. Ltd  -  Normal Situation

                    Final Management Plan

 I-------------------------------------------------------I
 I        COST               STKOUT            LABOUR    I
 I       dollars           av % short        % laid off  I
 I-------------------------------------------------------I
 I                                                       I
 I    $1,650,399.            15.87              5.69     I
 I                                                       I
 I-------------------------------------------------------I
#ET=3:57.4 PT=25.8 IO=22.2
```

A3.4.2 The Naive Method

Begins on the following page.

```
RUN MOLP/NAIVE
#RUNNING 5417
#?
Enter file to be read from (AUG__) plus fullstop,
AUGNM.
```

```
Enter your choice for solution values

COST     (  dollars    )
1538069
STKOUT   (av % short   )
6.36
LABOUR   ( % laid off )
0
```

```
    PLANNING MODEL for XYZ Co. Ltd  -  Normal Situation

                         COST           STKOUT        LABOUR
                         dollars      av % short    % laid off
                       ------------------------------------------

Your guess was.....   $1,538,069.      6.36%          0.00%

This guess was not feasible
 - consequently you will achieve less

Your solution is...   $1,648,536.     12.34%          6.73%

                       ------------------------------------------

Enter 1 if you wish to STOP , else enter 0
```

```
    PLANNING MODEL for XYZ Co. Ltd  -  Normal Situation

                       PREVIOUS  SOLUTION
 I-------------------------------------------------------I
 I       COST              STKOUT             LABOUR     I
 I      dollars          av % short         % laid off   I
 I-------------------------------------------------------I
 I                                                       I
 I   $1,648,536.            12.34              6.73      I
 I                                                       I
 I-------------------------------------------------------I

Enter your choice for solution values
COST     (  dollars    )
1600000
STKOUT   (av % short   )
14
LABOUR   ( % laid off )
7
```

```
   PLANNING MODEL for XYZ Co. Ltd  -  Normal Situation

                        COST         STKOUT       LABOUR
                       dollars     av % short   % laid off
                     -------------------------------------

Your guess was.....  $1,600,000.      14.00%        7.00%

This guess was not feasible
- consequently you will achieve less

Your solution is...  $1,619,894.      14.62%        7.87%

                     -------------------------------------
Enter 1 for all previous solutions, else enter 0
0
Enter 1 if you wish to STOP , else enter 0
0
```

```
    PLANNING MODEL for XYZ Co. Ltd  -  Normal Situation

                       PREVIOUS  SOLUTION
I--------------------------------------------------------I
I       COST              STKOUT             LABOUR      I
I      dollars          av % short         % laid off    I
I--------------------------------------------------------I
I                                                        I
I   $1,619,894.            14.62              7.87       I
I                                                        I
I--------------------------------------------------------I

Enter your choice for solution values
COST     (  dollars    )
1650000
STKOUT   (av % short   )
14.5
LABOUR   ( % laid off )
6.5
```

```
    PLANNING MODEL for XYZ Co. Ltd  -  Normal Situation

                              COST         STKOUT       LABOUR
                             dollars    av % short   % laid off
                         ----------------------------------------

Your guess was.....    $1,650,000.     14.50%         6.50%

This guess was feasible
- in fact you can achieve better

Your solution is...    $1,646,752.     14.37%         6.29%

                         ----------------------------------------

Enter 1 for all previous solutions, else enter 0
1
```

```
    PLANNING MODEL for XYZ Co. Ltd  -  Normal Situation

                              COST         STKOUT       LABOUR
                             dollars    av % short   % laid off
                         ----------------------------------------
Previous solution  1   $1,648,536.     12.34%         6.73%
Previous solution  2   $1,619,894.     14.62%         7.87%
Previous solution  3   $1,646,752.     14.37%         6.29%
                         ----------------------------------------
Enter 1 if you wish to STOP , else enter 0
0

Enter your choice for solution values
COST      (  dollars    )
1660000
STKOUT   (av % short   )
14
LABOUR   ( % laid off )
6
```

```
   PLANNING MODEL for XYZ Co. Ltd  -  Normal Situation

                              COST         STKOUT       LABOUR
                             dollars    av % short   % laid off
                         ----------------------------------------

Your guess was.....    $1,660,000.     14.00%         6.00%

This guess was feasible
 - in fact you can achieve better

Your solution is...    $1,657,128.     13.87%         5.79%

                         ----------------------------------------
Enter 1 for all previous solutions, else enter 0
0

Enter 1 if you wish to STOP , else enter 0
0
```

```
     PLANNING MODEL for XYZ Co. Ltd  -  Normal Situation

                        PREVIOUS  SOLUTION
 I-------------------------------------------------------I
 I       COST              STKOUT             LABOUR     I
 I      dollars          av % short         % laid off   I
 I-------------------------------------------------------I
 I                                                       I
 I   $1,657,128.           13.87               5.79      I
 I                                                       I
 I-------------------------------------------------------I

Enter your choice for solution values
COST     (  dollars    )
1650000
STKOUT   (av % short   )
14
LABOUR   ( % laid off )
6
```

```
   PLANNING MODEL for XYZ Co. Ltd  -  Normal Situation

                            COST       STKOUT      LABOUR
                           dollars   av % short  % laid off
                        -------------------------------------

Your guess was.....     $1,650,000.    14.00%       6.00%

This guess was not feasible
 - consequently you will achieve less

Your solution is...     $1,651,394.    14.06%       6.09%

                        -------------------------------------

Enter 1 for all previous solutions, else enter 0
1
```

```
    PLANNING MODEL for XYZ Co. Ltd  -  Normal Situation

                            COST       STKOUT       LABOUR
                           dollars   av % short   % laid off
                        --------------------------------------
Previous solution  1    $1,648,536.    12.34%       6.73%
Previous solution  2    $1,619,894.    14.62%       7.87%
Previous solution  3    $1,646,752.    14.37%       6.29%
Previous solution  4    $1,657,128.    13.87%       5.79%
Previous solution  5    $1,651,394.    14.06%       6.09%
                        --------------------------------------

Enter 1 if you wish to STOP , else enter 0
1
```

PLANNING MODEL for XYZ Co. Ltd - Normal Situation

Final Management Plan

COST dollars	STKOUT av % short	LABOUR % laid off
$1,651,394.	14.06	6.09

#ET=5:23.6 PT=5.4 IO=1.6

A3.4.3 The SWT Method

Begins on the following page.

```
RUN MOLP/SWTEXT
#RUNNING 5449
#?
Enter file to be read (SWT__) plus fullstop,
SWTNM.
Enter number of objective to be fixed
3
Enter level at which LABOUR is to be fixed
7.5
```

```
        SOLVING TO FIND SEVEN PAIRWISE SOLUTIONS
        ----------------------------------------
                     -Please  wait-
```

```
   PLANNING MODEL for XYZ Co. Ltd  -  Normal Situation

                   ........................................
  Plan                      COST                    STKOUT
  Number    1              dollars                av % short

                   $1,636,819.                      12.13%

                   ........................................

  Additional cost of reducing STKOUT
  by one unit is...                              $4,691.

Enter worth assessment in range -10 to +10
-2
```

```
   PLANNING MODEL for XYZ Co. Ltd  -  Normal Situation

                     .......................................
 Plan                      COST                   STKOUT
 Number    2             dollars                av % short

                  $1,663,043.                       8.57%

                     .......................................

 Additional cost of reducing STKOUT
 by one unit is...                               $8,796.

Enter worth assessment in range -10 to +10
-9
```

```
   PLANNING MODEL for XYZ Co. Ltd  -  Normal Situation

                     .......................................
 Plan                      COST                   STKOUT
 Number    3             dollars                av % short

                  $1,615,786.                      17.24%

                     .......................................

 Additional cost of reducing STKOUT
 by one unit is...                               $3,665.

Enter worth assessment in range -10 to +10
5
```

```
   PLANNING MODEL for XYZ Co. Ltd  -  Normal Situation

                     .......................................
 Plan                      COST                   STKOUT
 Number    4             dollars                av % short

                  $1,622,329.                      15.50%

                     .......................................

 Additional cost of reducing STKOUT
 by one unit is...                               $4,105.

Enter worth assessment in range -10 to +10
-2
```

PLANNING MODEL for XYZ Co. Ltd – Normal Situation

Plan Number 5	COST dollars	STKOUT av % short
	$1,630,598.	13.53%

Additional cost of reducing STKOUT by one unit is... $4,251.

Enter worth assessment in range -10 to +10
-1

PLANNING MODEL for XYZ Co. Ltd – Normal Situation

Plan Number 6	COST dollars	STKOUT av % short
	$1,644,152.	11.03%

Additional cost of reducing STKOUT by one unit is... $7,623.

Enter worth assessment in range -10 to +10
-4

PLANNING MODEL for XYZ Co. Ltd – Normal Situation

Plan Number 7	COST dollars	STKOUT av % short
	$1,673,914.	7.34%

Additional cost of reducing STKOUT by one unit is... $8,796.

Enter worth assessment in range -10 to +10
10

```
        Plot of Worth vs Solution Values for STKOUT
I07
I
I
I
I                                                          03
I
I
I
I                                                        Soln
I-------I---------I---------I---------I---------I--------I
I                                      05                 Val
I                               01               04
I
I                      06
I
I
I
I      02
I
```

```
      Solution Values                 Tradeoff       Worth
       COST          STKOUT           Value($)     Assessment

-----------------------------------------------------------
3   $1,615,786.     17.24%            $3,665.          5.0
4   $1,622,329.     15.50%            $4,105.         -2.0
5   $1,630,598.     13.53%            $4,251.         -1.0
1   $1,636,819.     12.13%            $4,691.         -2.0
6   $1,644,152.     11.03%            $7,623.         -4.0
2   $1,663,043.      8.57%            $8,796.         -9.0
7   $1,673,914.      7.34%            $8,796.         10.0

Satisfied with your assessments? Enter 1 if YES, else 0
0
Enter the number of changes to be made
2
Enter plans to be changed - separate by commas
4,7
```

```
   PLANNING MODEL for XYZ Co. Ltd  -  Normal Situation

                  ........................................
  Plan                   COST                   STKOUT
  Number    4           dollars              av % short

                   $1,622,329.                  15.50%

                  ........................................

  Additional cost of reducing STKOUT
  by one unit is...                          $4,105.

Enter worth assessment in range -10 to +10
2
```

PLANNING MODEL for XYZ Co. Ltd - Normal Situation

Plan Number 7	COST dollars	STKOUT av % short
	$1,673,914.	7.34%

Additional cost of reducing STKOUT
by one unit is... $8,796.

Enter worth assessment in range -10 to +10
-10

```
    Plot of Worth vs Solution Values for STKOUT
I
I
I
I
I                                                       03
I
I
I                                              04
I                                                    Soln
I-------I---------I---------I---------I---------I--------I
I                                    05               Val
I                            01
I
I                    06.
I
I
I
I      02
I07
```

	Solution Values COST	STKOUT	Tradeoff Value($)	Worth Assessment
3	$1,615,786.	17.24%	$3,665.	5.0
4	$1,622,329.	15.50%	$4,105.	2.0
5	$1,630,598.	13.53%	$4,251.	-1.0
1	$1,636,819.	12.13%	$4,691.	-2.0
6	$1,644,152.	11.03%	$7,623.	-4.0
2	$1,663,043.	8.57%	$8,796.	-9.0
7	$1,673,914.	7.34%	$8,796.	-10.0

Satisfied with your assessments? Enter 1 if YES, else 0
1
The regression solution is 13.97%
Enter your own value or zero for no change
14

SOLVING TO FIND SEVEN PAIRWISE SOLUTIONS

-Please wait-

PLANNING MODEL for XYZ Co. Ltd - Normal Situation

Plan Number 1	COST dollars	LABOUR % laid off
	$1,659,632.	5.61%

Additional cost of reducing LABOUR by one unit is... $17,958.

Enter worth assessment in range -10 to +10
-2

PLANNING MODEL for XYZ Co. Ltd - Normal Situation

Plan Number 2	COST dollars	LABOUR % laid off
	$1,690,092.	3.92%

Additional cost of reducing LABOUR by one unit is... $17,958.

Enter worth assessment in range -10 to +10
-7

```
   PLANNING MODEL for XYZ Co. Ltd  -  Normal Situation

                  .......................................
 Plan                    COST                    LABOUR
 Number   3             dollars                % laid off

                 $1,613,154.                      8.44%

                  .......................................

 Additional cost of reducing LABOUR
 by one unit is...                             $16,396.

Enter worth assessment in range -10 to +10
3
```

```
   PLANNING MODEL for XYZ Co. Ltd  -  Normal Situation

                  .......................................
 Plan                    COST                    LABOUR
 Number   4             dollars                % laid off

                 $1,719,832.                      2.32%

                  .......................................

 Additional cost of reducing LABOUR
 by one unit is...                             $19,520.

Enter worth assessment in range -10 to +10
-10
```

```
   PLANNING MODEL for XYZ Co. Ltd  -  Normal Situation

                  .......................................
 Plan                    COST                    LABOUR
 Number   5             dollars                % laid off

                 $1,574,613.                     11.07%

                  .......................................

 Additional cost of reducing LABOUR
 by one unit is...                             $13,664.

Enter worth assessment in range -10 to +10
8
```

```
    PLANNING MODEL for XYZ Co. Ltd  -  Normal Situation

                ........................................
  Plan                    COST                  LABOUR
  Number   6             dollars              % laid off

                 $1,548,484.                     12.98%

                ........................................

  Additional cost of reducing LABOUR
  by one unit is...                          $13,664.

Enter worth assessment in range -10 to +10
10
```

```
    PLANNING MODEL for XYZ Co. Ltd  -  Normal Situation

                ........................................
  Plan                    COST                  LABOUR
  Number   7             dollars              % laid off

                 $1,759,912.                      0.26%

                ........................................

  Additional cost of reducing LABOUR
  by one unit is...                          $19,520.

Enter worth assessment in range -10 to +10
-10
```

```
        Plot of Worth vs Solution Values for LABOUR
I                                                      06
I
I                                                05
I
I
I
I                                    03
I
I                                                     Soln
I--------I---------I---------I---------I---------I--------I
I                                                      Val
I                        01
I
I
I
I                02
I
I
I07      04
```

```
        Solution Values                    Tradeoff        Worth
        COST              LABOUR           Value($)      Assessment

----------------------------------------------------------------------

6   $1,548,484.        12.98%           $13,664.          10.0
5   $1,574,613.        11.07%           $13,664.           8.0
3   $1,613,154.         8.44%           $16,396.           3.0
1   $1,659,632.         5.61%           $17,958.          -2.0
2   $1,690,092.         3.92%           $17,958.          -7.0
4   $1,719,832.         2.32%           $19,520.         -10.0
7   $1,759,912.         0.26%           $19,520.         -10.0

Satisfied with your assessments? Enter 1 if YES, else 0
1
The regression solution is      7.02%
Enter your own value or zero for no change
6.5
```

```
      PLANNING MODEL for XYZ Co. Ltd  -  Normal Situation

                    Final Management Plan

 I-------------------------------------------------------I
 I        COST             STKOUT            LABOUR      I
 I       dollars         av % short        % laid off    I
 I-------------------------------------------------------I
 I                                                       I
 I   $1,644,976.           14.00               6.50      I
 I                                                       I
 I-------------------------------------------------------I
#ET=7:49.3 PT=16.6 IO=2.4
```

A3.4.4 The STE Method

Begins on the following page.

```
RUN MOLP/STE
#RUNNING 3089
#?
Enter name of data file (AUG__) plus fullstop,
AUGNM.
```

```
SOLVING TO FIND SIX SOLUTIONS
-----------------------------
-Please wait-
```

PLANNING MODEL for XYZ Co. Ltd - Normal Situation

Solution number	COST dollars	STKOUT av % short	LABOUR % laid off
1	$1,696,160.	10.95%	4.48%
2	$1,598,247.	12.74%	9.78%
3	$1,737,609.	17.24%	0.75%
4	$1,643,547.	6.36%	10.14%
5	$1,539,833.	16.84%	12.80%
6	$1,632,393.	19.85%	6.13%

```
Enter the number of the solution you most prefer.
      0 implies the previous solution.
2
```

Enter 1 if you are satisfied and want to stop, else 0
0

SOLVING TO FIND SIX SOLUTIONS

-Please wait-

PLANNING MODEL for XYZ Co. Ltd - Normal Situation

Solution number	COST dollars	STKOUT av % short	LABOUR % laid off
1	$1,626,378.	6.69%	11.13%
2	$1,584,058.	10.79%	11.73%
3	$1,638,933.	15.13%	6.58%
4	$1,666,677.	10.76%	6.25%
5	$1,619,616.	11.67%	8.70%
6	$1,589,730.	16.60%	9.25%
Previous Solution	$1,598,247.	12.74%	9.78%

Enter the number of the solution you most prefer.
0 implies the previous solution.
0

Enter 1 if you are satisfied and want to stop, else 0
0

SOLVING TO FIND SIX SOLUTIONS

-Please wait-

PLANNING MODEL for XYZ Co. Ltd - Normal Situation

Solution number	COST dollars	STKOUT av % short	LABOUR % laid off
1	$1,615,323.	11.68%	8.96%
2	$1,588,509.	15.15%	9.72%
3	$1,614,517.	14.15%	8.32%
4	$1,567,610.	11.99%	12.31%
5	$1,603,245.	9.54%	11.03%
6	$1,586,567.	11.98%	10.93%
Previous Solution	$1,598,247.	12.74%	9.78%

Enter the number of the solution you most prefer.
0 implies the previous solution.

3

Enter 1 if you are satisfied and want to stop, else 0

0

SOLVING TO FIND SIX SOLUTIONS

-Please wait-

PLANNING MODEL for XYZ Co. Ltd - Normal Situation

Solution number	COST dollars	STKOUT av % short	LABOUR % laid off
1	$1,606,816.	14.01%	8.83%
2	$1,631,934.	14.56%	7.15%
3	$1,596,182.	15.06%	9.23%
4	$1,624,948.	12.16%	8.22%
5	$1,622,374.	15.84%	7.41%
6	$1,607,366.	12.25%	9.30%
Previous Solution	$1,614,517.	14.15%	8.32%

Enter the number of the solution you most prefer.
0 implies the previous solution.

2

```
 Enter 1 if you are satisfied and want to stop, else 0
0



                  SOLVING TO FIND SIX SOLUTIONS
                  -----------------------------
                         -Please  wait-
```

PLANNING MODEL for XYZ Co. Ltd - Normal Situation

Solution number	COST dollars	STKOUT av % short	LABOUR % laid off
1	$1,626,612.	14.75%	7.43%
2	$1,635,227.	13.60%	7.20%
3	$1,642,391.	14.16%	6.62%
4	$1,632,951.	15.46%	6.86%
5	$1,621,699.	13.82%	7.97%
6	$1,642,945.	15.15%	6.33%
Previous Solution	$1,631,934.	14.56%	7.15%

```
 Enter the number of the solution you most prefer.
       0 implies the previous solution.
3
```

```
 Enter 1 if you are satisfied and want to stop, else 0
0



                  SOLVING TO FIND SIX SOLUTIONS
                  -----------------------------
                         -Please  wait-
```

PLANNING MODEL for XYZ Co. Ltd - Normal Situation

Solution number	COST dollars	STKOUT av % short	LABOUR % laid off
1	$1,643,210.	14.18%	6.56%
2	$1,644,600.	13.70%	6.60%
3	$1,637,526.	14.13%	6.92%
4	$1,636,953.	14.45%	6.87%
5	$1,650,831.	13.71%	6.22%
6	$1,648,935.	14.53%	6.12%
Previous Solution	$1,642,391.	14.16%	6.62%

Enter the number of the solution you most prefer.
0 implies the previous solution.

0

Enter 1 if you are satisfied and want to stop, else 0

1

PLANNING MODEL for XYZ Co. Ltd - Normal Situation

Final Management Plan

COST dollars	STKOUT av % short	LABOUR % laid off
$1,642,391.	14.16	6.62

#ET=6:59.2 PT=59.9 IO=10.5

A3.5 QUESTIONNAIRES

This final section of the appendix contains the two questionnaires used for externalizing subjective assessments from each subject. The first questionnaire was given upon completing each solution method, and the second one was given at the completion of all four solution methods. Subjects were required to place a mark on the appropriate scale. These marks were later measured and scaled between Ø and 1Ø. The resulting statistical analysis was based on these measured scores, and the rankings derived from them.

Questionnaire 1

Name: Date:

Method:

1. Confidence in your final solution.

-very dissatisfied
-am sure I could do better

-very satisfied
-do not think I could do better

2. Ease of use of the method.

-very difficult to use

-very easy to use

3. Ease of understanding the logic of the method.

-very unclear as to how I got my final solution

-understand clearly as to how I got to my final solution

Questionnaire 2

Name:

Date:

FINAL QUESTION

Recall the four methods you have used:

-ZW	comparison of close solutions
-Naive	process of guessing with some feedback
-SWT	assessing pairwise tradeoffs
-STE	choosing among solutions and shrinking down

Consider the situation where you have to actually use one of these methods to find an acceptable solution to a similar mathematical model having at least three objectives.

Mark four points on the scale below which reflect your relative preference for using each of the methods in a practical situation.

extremely hesitant to use	relatively indifferent	very willing to use

A3.6 THE RAW DATA

This section contains all the raw data of the experiment.

A3.6.1 Scores for each Criterion (for the 24 subjects)

	Confidence				Ease of Use			
	ZW	Naive	SWT	STE	ZW	Naive	SWT	STE
1	5.0	5.1	5.9	8.5	5.2	6.1	4.8	9.0
2	3.1	7.9	7.8	7.1	8.1	9.2	8.9	7.8
3	1.4	8.2	5.7	8.5	9.4	6.2	8.1	8.5
4	3.4	8.3	8.8	8.7	6.2	7.6	2.4	5.4
5	6.2	10.0	4.6	5.8	7.9	9.1	2.5	9.3
6	4.7	7.6	8.8	7.7	10.0	10.0	7.7	10.0
7	10.0	6.6	8.4	10.0	10.0	10.0	7.2	10.0
8	2.2	4.8	9.0	7.5	8.2	8.0	5.0	3.2
9	2.0	8.6	4.6	6.0	9.4	6.6	3.4	7.6
10	4.6	3.8	7.3	7.6	5.0	7.8	1.0	9.4
11	1.8	2.1	2.4	9.3	7.2	2.9	8.3	8.3
12	3.2	9.1	7.6	7.6	6.9	3.3	8.3	7.2
13	5.6	3.8	6.9	7.8	9.4	8.8	3.5	7.4
14	3.1	5.5	6.6	5.9	2.5	5.9	4.8	5.7
15	5.9	8.3	8.8	8.4	5.9	6.4	7.8	7.9
16	2.2	2.0	1.2	6.1	8.4	9.1	2.9	8.7
17	6.1	6.7	8.8	7.6	9.4	8.2	5.0	9.7
18	4.8	7.8	3.5	6.1	9.3	9.4	9.5	9.4
19	2.9	6.6	5.2	8.0	4.9	7.8	4.8	8.8
20	0.6	6.9	7.4	9.8	9.1	9.6	9.9	10.0
21	7.8	8.8	9.0	9.6	9.0	9.6	7.0	9.3
22	2.4	10.0	9.3	10.0	7.5	7.9	7.8	9.8
23	3.4	7.7	7.4	5.0	4.8	7.2	5.9	6.4
24	6.6	5.9	3.5	7.5	7.4	7.5	3.4	9.1

	Ease of Understanding				CPU Time (secs)			
	ZW	Naive	SWT	STE	ZW	Naive	SWT	STE
1	8.0	9.4	4.4	9.0	25.8	5.0	18.2	85.3
2	8.7	9.4	9.4	8.5	10.4	5.1	14.7	64.5
3	8.4	7.4	6.3	8.2	6.3	7.2	17.7	68.8
4	7.5	7.8	7.0	8.6	24.8	6.5	16.4	65.0
5	6.4	10.0	7.2	10.0	10.7	4.3	16.6	40.7
6	10.0	10.0	9.4	10.0	15.5	7.1	16.8	102.4
7	10.0	10.0	3.7	10.0	21.0	9.9	17.3	76.4
8	9.0	8.6	7.4	10.0	23.4	6.8	16.1	70.9
9	9.4	9.2	9.0	8.8	15.8	8.1	17.5	75.4
10	3.5	6.7	3.6	9.4	17.0	8.1	15.3	55.0
11	1.3	9.8	8.2	9.0	23.0	6.3	15.7	67.7
12	3.6	5.1	8.5	7.4	26.1	9.4	18.1	48.2
13	7.8	8.0	5.9	7.8	20.1	2.9	18.2	74.9
14	7.0	8.5	7.6	9.1	26.2	5.8	15.7	62.9
15	6.6	7.9	9.1	9.8	21.0	5.6	17.3	46.0
16	7.8	4.6	7.7	5.5	22.9	7.2	15.4	66.4
17	9.1	5.8	4.8	9.6	5.6	4.7	17.6	28.9
18	9.5	9.6	9.3	9.1	15.9	4.9	15.5	71.4
19	4.6	7.5	5.0	8.5	23.5	6.5	19.2	51.1
20	2.9	9.8	9.6	10.0	15.3	6.0	16.5	52.5
21	7.0	9.6	9.0	9.0	10.1	5.8	16.0	38.8
22	9.6	7.0	9.6	10.0	11.0	7.5	18.5	29.9
23	8.5	8.6	8.5	7.8	17.8	6.0	17.9	54.9
24	8.3	9.3	6.1	6.2	20.0[a]	3.8	20.6	36.5

	Elapsed Time (mins)				PREFERENCE FOR USE			
	ZW	Naive	SWT	STE	ZW	Naive	SWT	STE
1	9.68	8.73	27.00	12.68	3.03	1.52	5.61	8.03
2	2.64	5.28	11.72	9.91	3.01	8.49	7.21	8.08
3	1.48	7.72	7.80	5.61	5.48	1.40	8.15	6.69
4	4.83	10.50	13.20	6.75	0.39	8.80	0.64	4.01
5	5.28	5.57	20.70	7.28	5.73	8.56	2.19	6.52
6	4.28	7.41	10.65	10.71	5.73	6.22	8.03	8.85
7	6.67	17.40	10.94	9.24	10.00	1.91	0.96	7.77
8	8.57	12.26	28.79	18.07	2.23	9.17	5.92	2.68
9	9.15	18.23	21.34	14.73	1.15	8.85	3.63	8.30
10	6.48	12.32	19.33	6.63	3.57	8.15	2.55	9.36
11	5.79	11.97	18.18	14.88	2.68	0.99	7.20	9.78
12	5.47	10.96	13.14	9.70	2.68	6.37	8.05	7.39
13	5.31	5.89	13.27	7.47	5.76	1.53	6.88	8.74
14	8.41	10.06	20.19	9.31	1.27	4.69	6.57	7.54
15	5.28	6.60	16.74	8.84	1.53	8.28	3.44	8.79
16	6.92	15.81	18.90	6.94	5.92	0.25	1.91	8.79
17	2.55	9.16	16.69	5.88	7.26	0.96	3.95	9.30
18	4.63	8.57	18.25	13.93	4.74	7.96	3.18	6.22
19	8.24	14.28	17.86	9.13	7.32	8.47	5.80	6.62
20	3.30	11.51	23.35	8.23	2.87	7.96	8.66	9.62
21	4.64	11.95	19.65	7.57	9.49	2.93	4.97	8.73
22	9.45	26.00	25.53	9.12	0.76	8.66	5.09	8.02
23	4.74	6.65	12.47	6.52	5.10	8.30	6.62	3.75
24	5.00[a]	4.98	16.70	5.25	7.90	5.92	2.29	8.66

a - These two observations were not immediately recorded; they were therefore estimated from memory.

A3.6.2 Position of the Solution Method

i.e., 1st, 2nd, 3rd or 4th

	ZW	Naive	SWT	STE
1	1	4	2	3
2	2	3	1	4
3	3	2	4	1
4	4	1	3	2
5	2	3	4	1
6	1	4	3	2
7	4	1	2	3
8	3	2	1	4
9	1	2	3	4
10	2	1	4	3
11	3	4	1	2
12	4	3	2	1
13	1	3	2	4
14	2	4	1	3
15	4	2	3	1
16	3	1	4	2
17	2	4	3	1
18	1	3	4	2
19	3	1	2	4
20	4	2	1	3
21	1	2	4	3
22	2	1	3	4
23	3	4	1	2
24	4	3	2	1

Note: These figures should represent the 24 possible permutations of four digits, however rows 23 and 24 are identical to rows 11 and 12.

APPENDIX 4

This appendix contains two sets of data; all 55 extreme points of the MOLP for the normal planning situation and a set of 16 randomly generated plans, also from the normal situation MOLP, which are used in Section 6.2 to illustrate the SJT method.

1 The 55 Efficient Extreme Point Solutions

The utility values are based on the following utility function

$$\underline{U} = -[\ 4.0E-4\ (f_1-1740797) + 2.0\ f_2 + 6.0\ f_3\ ]\ .$$

The solutions have been sorted into descending utility.

	COST	STKOUT	LABOUR	Utility
1	$1,614,330	11.769%	8.974%	-26.795
2	$1,609,650	12.766%	8.974%	-26.917
3	$1,590,605	13.117%	10.190%	-27.297
4	$1,588,850	13.949%	10.040%	-27.359
5	$1,601,168	14.762%	8.974%	-27.516
6	$1,586,218	15.611%	9.740%	-27.830
7	$1,596,618	15.870%	8.974%	-27.912
8	$1,584,918	16.535%	9.574%	-28.162
9	$1,569,513	11.769%	12.254%	-28.548
10	$1,668,111	11.769%	5.694%	-28.627
11	$1,663,431	12.766%	5.694%	-28.749
12	$1,545,788	13.117%	13.470%	-29.050
13	$1,544,033	13.949%	13.320%	-29.112
14	$1,654,949	14.762%	5.694%	-29.348
15	$1,586,462	18.641%	8.974%	-29.392
16	$1,582,887	18.845%	9.158%	-29.474
17	$1,541,400	15.611%	13.020%	-29.583
18	$1,650,399	15.870%	5.694%	-29.744
19	$1,540,100	16.535%	12.854%	-29.915
20	$1,637,568	8.720%	8.974%	-29.992
21	$1,640,243	18.641%	5.694%	-31.224
22	$1,538,069	18.845%	12.438%	-31.227
23	$1,636,668	18.845%	5.877%	-31.300
24	$1,585,747	19.861%	8.974%	-31.546
25	$1,592,750	8.720%	12.254%	-31.745
26	$1,691,349	8.720%	5.694%	-31.824
27	$1,717,446	11.769%	2.947%	-31.879
28	$1,712,766	12.766%	2.947%	-32.001
29	$1,704,284	14.762%	2.947%	-32.600
30	$1,551,150	11.085%	14.386%	-32.627
31	$1,653,168	6.947%	8.974%	-32.686
32	$1,699,734	15.870%	2.947%	-32.997
33	$1,639,528	19.861%	5.694%	-33.378
34	$1,655,768	6.355%	9.241%	-34.144
35	$1,608,350	6.947%	12.254%	-34.439
36	$1,689,578	18.641%	2.947%	-34.476

37	$1,706,949	6.947%	5.694%	-34.519
38	$1,686,003	18.845%	3.130%	-34.552
39	$1,740,684	8.720%	2.947%	-35.077
40	$1,610,950	6.355%	12.521%	-35.897
41	$1,709,549	6.355%	5.961%	-35.977
42	$1,688,863	19.861%	2.947%	-36.630
43	$1,558,950	9.311%	15.185%	-36.993
44	$1,774,971	11.769%	0.000%	-37.208
45	$1,770,291	12.766%	0.000%	-37.330
46	$1,756,284	6.947%	2.947%	-37.771
47	$1,761,809	14.762%	0.000%	-37.929
48	$1,757,259	15.870%	0.000%	-38.325
49	$1,787,500	10.125%	0.000%	-38.931
50	$1,758,884	6.355%	3.213%	-39.223
51	$1,787,500	8.720%	0.549%	-39.415
52	$1,747,103	18.641%	0.000%	-39.804
53	$1,787,500	6.947%	1.348%	-40.663
54	$1,787,500	6.355%	1.747%	-41.873
55	$1,746,387	19.861%	0.000%	-41.958

2 The 16 Plans (non-extreme) as used in Section 6.2

	COST	STKOUT	LABOUR
1	$1,635,240	13.710%	7.166%
2	$1,752,296	16.238%	0.183%
3	$1,698,051	6.355%	6.659%
4	$1,724,757	18.236%	1.218%
5	$1,620,585	11.352%	8.783%
6	$1,718,858	8.125%	4.451%
7	$1,538,225	18.610%	12.480%
8	$1,595,016	7.258%	13.335%
9	$1,563,461	10.899%	13.396%
10	$1,626,008	8.744%	9.803%
11	$1,651,674	16.363%	5.520%
12	$1,571,023	18.198%	10.180%
13	$1,588,899	13.324%	10.242%
14	$1,684,270	14.288%	4.171%
15	$1,749,678	6.355%	3.723%
16	$1,670,116	19.843%	3.989%